ESSAYS ON TRADE LIBERALIZATION WITH FIRM HETEROGENEITY

By

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To my son, Peter

To my daughter, Marta

and

To my wife, Tatiana

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INTRODUCTION

This study responds to important considerations prior to implementing trade liberalization. We consider how trade liberalization influences the distribution of firms over three dimensions: productivity, size (amounts of employed factors), and collected revenue/profit. Specifically, we look at the spillover effects of trade liberalization in one sector on the average productivity of firms in the other sector. We study how trade liberalization affects the number of firms (and a share of exporting firms) in different sectors. Finally, we analyze how trade liberalization leads to short-run changes in the welfare of owners of different factors used in production and the reallocation of factors across sectors.

The short-run effects of changes in trade policy on the owners of different production factors in a small economy are often analyzed using the specific-factors model (Jones, 1971; Mayer,1974; Mussa, 1974; Neary, 1978). The specific factor model is a two sector model in which each sector produces a homogeneous good using a sector specific factor and a factor that is mobile between sectors. The prices of goods, produced in both sectors, are exogenously given, since the assumption is that the country's economy is small, relative to the economy of the rest of the world. Perfect competition is assumed to be the market structure in both sectors of the model. Jones, 1971, established the magnification effect of the changes in commodity prices with respect to the prices of sector-specific factors. Particularly, with percentage changes in sector prices: $\widehat{p}_1$, $\widehat{p}_2$, changes in factor prices satisfy to $\widehat{r}_1 > \widehat{p}_1 > w > \widehat{p}_2 > \widehat{r}_2$. Trade liberalization leads to an increase in the ratio of domestic price of the exported commodity to domestic price of imported commodity. This increases the ratio of rental on capital to the commodity price in the exporting sector and decreases the ratio of rental on capital to the commodity price in the importing sector. As result, the owner of capital in the exporting sector can buy more of both goods, and the owner of capital in the importing sector can buy less of both goods. The owners of labor can buy more of the imported good and less of the exported good, as the percentage change in wage rate is bounded by the percentage changes in commodity prices. Whether or not labor owner's welfare increases or decreases depends on the share of the imported good in consumption. As trade liberalizes, labor moves partially from the importing sector to the exporting sector.

Even though the traditional specific-factors framework is used often for the analysis of the short-run effects of trade liberalization, it can not account for some stylized facts about international trade. One stylized fact suggests substantial intra-industry trade among industrialized countries that has grown over time (Balassa, 1966, and Grubel, 1967). In the recent literature, this fact finds support as well. For example, Helpman, 1999, points out that the share of intra-industry trade among for many European countries increased substantially between 1970 and 1990. This fact can not be explained within the traditional framework (specific-factors model), as no place exists for two-way trade within sector producing a homogeneous good. Moreover the gravity equation, that performs well in data, could be justified theoretically through monopolistic competition market structure, that is usually used to model intra-industry trade (Bergstrand, 1989).

Krugman, 1979, addressed intra-industry trade in a one sector model with monopolistic competition market structure. In this framework every firm, though employing the same technology, produces different variety. Since there are many varieties on the market, the changes in the price of one variety have no effect on the demand for another variety. In this sense, the firm, setting the price for the variety it produces, behaves as a monopolist. It happens that in transition from autarky to free trade, the price of any variety relative to the wage rate decreases. Moreover, the number of varieties increases with the transition from autarky to free trade.

Krugman, 1981, has the framework with two sectors, two countries and the monopolistic competition market structure in both sectors. The model has sector specific factors only and no mobile factors. Krugman, 1981, found that in the comparative disadvantage sector, the return to the fixed factor decreases with trade liberalization (transition from autarky to free trade). At the same time, in the comparative advantage sector, the return to the fixed factor increases with trade liberalization. Undoubtedly, the owner of the factor in comparative advantage sector is better off with trade liberalization. The owner of the factor in comparative disadvantage sector can be better off or worth off with transition from autarky to free trade. If the elasticity of the demand is smaller than certain threshold, then the owner of the scarce factors is better off in course of transition from autarky to free trade. For the elasticity of demand above this threshold, the owner of scarce factors is better off if the factor proportions are similar. And the owner of scarce factor becomes worth off with trade liberalization if the factor proportions are more different.

It is worth to compare Krugman, 1981, framework with the traditional specific factor model. Let's point out the difference between these models in first place. Krugman, 1981, model is two countries, two sectors model with sector specific factors only. The traditional specific factors model is the small open economy model with two sector specific factors and one mobile factor. The principal difference between these models is defined by the market structure. Traditional sector specific model has perfect competition market structure, while Krugman, 1981, framework has monopolistic competition market structure.

With trade liberalization (decrease in trade costs), the ratio of domestic price of the exported commodity to domestic price of imported commodity increases in the traditional specific factors model. Also, the ratio of the price of any variety in comparative advantage sector to the price of any variety in comparative disadvantage sector increase with trade liberalization (transition from autarky to free trade) in Krugman, 1981, framework. The difference from traditional specific factors model is that in every sector a country imports some varieties and exports the varieties produced domestically. At the same time, country becomes net exporter in comparative advantage sector and net importer in comparative disadvantage sector. So, the price of any variety in the sector, where country will be net exporter, relative to the price of any variety in the sector, where country will be net importer, increases in Krugman, 1981, framework.

We have the magnification effect, which is similar to the one in the traditional specific factors model, in Krugman, 1981, framework. Specifically, with transition from autarky to free trade the ratio of the return to sector specific factor to the price of any variety within the same sector increase in comparative advantage sector and decreases in comparative disadvantage sector. As result, the owner of the factor of production in comparative advantage sector is able to buy more of every variety he/she consumed before trade liberalization. In addition to this effect, the number of available for consumption varieties increase with transition from autarky to free trade. At the same time, the owner of the factor of production in comparative disadvantage sector will be able to purchase less of every variety he/she consumed before trade liberalization. But, the number of available varieties increases with trade liberalization. The increase in the number of available for consumption varieties (variety effect) can compensate the negative magnification effect in comparative disadvantage sector. The variety effect is larger, the smaller is the elasticity of demand and more similar are the factor proportions.

Another stylized fact for which the traditional approach does not account is the existence of considerable heterogeneity of firms with respect to productivity. The considerable heterogeneity of firms with respect to productivity is one of the features of the international trade system, and some of the studies have provided insights into the behavior of firms, depending on their productivity. Clerides, Lach, & Tybout, 1998, did not find the evidence in the support of the fact that exporting might cause improvements in productivity because of learning by exporting. Conversely, firms with high productivity self-select themselves for exporting. Also, Bernard, & Jensen, 1999, support the fact that firms self-select themselves into exporting. Consequently, Aw, Chung, & Roberts, 2000, showed that trade liberalization forces the least productive firms to exit the market.

Both these stylized facts have been addressed in Melitz, 2003. Melitz, 2003, has introduced heterogeneous firms on the top of monopolistic competition market structure by Krugman, 1979, in one sector model with many countries. He found that with trade liberalization the average productivity of firms increase, since less productive firms leave the market. In this case, trade liberalization (the decrease in the fixed trade cost or the decrease in the variable trade cost) leads to the increase in the value of the smallest productivity among the firms on the market.

Bond, 1986, introduces the heterogeneity of firms with respect to productivity in the setup of the two sector model with two mobile factors, keeping the small economy assumption. In Bond, 1986, setup, price taking firms produce homogenous commodity. Firms are associated with entrepreneurs they are run by. And the entrepreneurial ability defines the firm's productivity. The firm with smallest productivity on the market is the one making the profit that is equal to the wage rate earned by entrepreneur when he is employed by any other firm. Since, firms produce the homogeneous commodity, there is no subdivision of firms into exporters and non-exporters as well as intra-industry trade is not modeled.

Bernard, Redding, & Schott, 2007, extended Melitz, 2003, framework to two sectors model that has two countries and two mobile factors of production in both countries. Or, equivalently, they extended Heckscher-Ohlin model with two countries by changing the market structure from perfect competition to monopolistic competition with heterogeneous firms. They analyzed in detail the transition from autarky to costly trade state with the fixed trade cost, variable trade cost and fixed production cost being the same across sectors. They found that average productivity of firms increases in both sectors with transition from autarky to costly trade. Moreover, the average productivity increases more in comparative advantage sector than in comparative disadvantage sector. In a sense they found that the exogenous comparative advantage is magnified by the changes in average productivity of firms in a course of transition from autarky to costly trade. Also, Bernard, Redding, & Schott, 2007, found that the average productivity of firms exporting some of their output abroad decreases more in comparative advantage sector than in comparative disadvantage sector. While adding to the standard model with two countries and two sectors, having the mobile factors of production, monopolistic competition market structure with heterogeneous firms, Bernard, Redding, & Schott, 2007, found that the relative nominal reward of abundant factor rises and relative nominal reward of scarce factor fall in the course of transition from autarky to costly trade. So, changing the market structure of the standard model from perfect competition to monopolistic competition with heterogenous firms does not alter the results on the direction of the changes in the relative nominal reward of factors of production.

We study the effect of trade liberalization (reduction in variable trade cost) in the sector specific factors model with two countries, that has monopolistic competition market structure with heterogeneous firms at least in one of the sectors. Particularly, the effect of the trade liberalization (the reduction in trade costs) in one sector on average productivity of the firms in the other sector has not been analyzed before. We would like to stress that the decrease in variable trade cost, while being at costly trade state, is the type of trade liberalization we analyze. This is very realistic case, since relatively few countries will experience the transition from autarky to costly trade (the type of trade liberalization analyzed in Bernard, Redding, & Schott, 2007) in foreseeable future. In the sector 1, we have monopolistic competition market structure with heterogenous firms of Melitz, 2003, type. We are exploring the effect of the reduction in trade cost in sector 2 on average productivity of firms in sector 1 across countries as well as on the average productivity of exporting firms in sector 1 across countries. In addition, we explore the changes in the return to the factors of production when trade costs decrease in sector 2 in two countries, two sectors model with monopolistic competition market structure and heterogeneous firms in sector 1.

We explore two setups of the model in detail. In chapter I, we assume that the market structure of sector 2 is the one of perfect competition. This case corresponds to the reduction of trade costs in the sector with homogeneous commodity and perfect competition market structure. The agricultural sector is a good example of the sector with perfect competition market structure and homogeneous commodity. Because of trade liberalization, the trade costs have been reduced substantially in the number of sectors in the recent history. The tariffs in agricultural sector have not been reduced substantially. At the same time, the negotiation on tariff reduction in the agricultural sector is in progress. Analyzing the decrease in the trade costs in sector 2 with the perfect competition market structure and homogeneous good allows for the predictions about the effects of potential trade liberalization in agricultural sector on average productivity of firms in the other sectors as well as other variables of interest. Moreover, the changes in trade costs associated with the changes in transportations costs could be analyzed in this framework as well.

The changes in trade costs influence the average productivity of firms within sector 1 of every country as well as the average productivity of exporting firms there. The increase in the average productivity of firms within sector 1 of particular country is caused by the exit of the firms with very low productivity. Similarly, the increase in the average productivity of exporting firms in sector 1 of particular country is caused by the exit of the exporting firms with low productivity from foreign market. The decrease in the average productivity of firms is caused by the successful entry of the firms with productivity smaller than the productivity of the least productive firm in the steady state before the changes in the trade costs. Similarly, the decrease in the average productivity of exporting firms is caused by the successful entry to the foreign market of the firms with productivity smaller than the productivities of exporting firms before trade liberalization.

The main contribution of this work is that it provides the cross-sectorial effects of the trade liberalization in one sector on the average productivity of firms (exporting firms) in the other sector in each country. It is interesting that the effect of trade liberalization in sector 2 on the average productivity of firms in sector 1 of particular country depends on whether the sector 1 of this country is of comparative advantage or of comparative disadvantage. In the case, the country has the comparative disadvantage in sector 1, the average productivity of firms in sector 1 there decreases with trade liberalization in sector 2. While the average productivity of exporting firms in the sector 1 of this country increases in this case. Conversely, if the country has comparative advantage in sector 1, the average productivity of firms in sector 1 of this country, increases with trade liberalization in sector 2. And the average productivity of exporting firms in sector 1 of this country decreases in this case.

In addition to these new findings, we state that the return to sector specific capital rises in comparative advantage sector and decreases in comparative disadvantage. This result agrees with the predictions of two countries, two sectors specific factors model, when both sectors have the perfect competition market structure. Also, the average productivity of firms in sector 1 of each country decreases in response to the decrease in the variable trade cost in sector 1. And the average productivity of exporting firms within sector 1 of every country decreases with the decrease in the variable trade cost in this sector. This observation again agrees with Melitz, 2003.

In chapter II, we assume that the market structure of sector 2 is of monopolistic competition with heterogeneous firms as the one in sector 1. This framework allows for the analysis of the effect of trade liberalization in the sector with differentiated commodity on the other sectors with differentiated commodities. The modified framework is used for exploring the mechanism of the effect of trade liberalization in one sector on the average productivity of firms (exporting firms) in the other sector, when both sectors are of monopolistic competition market structure with heterogeneous firms. This framework enables the analysis of trade liberalization in the apparel sector on soft drinks industry. We have analyzed the specific case of this framework, when the comparative advantage is driven by the differences in sector specific capital. In this case, the results about the spillover effect of trade liberalization on average productivity of the firms in the other sector of particular country do not change from the case when market structure differs across sectors (perfect competition in one sector and monopolistic competition with heterogeneous firms in the other).

CHAPTER I

TRADE LIBERALIZATION IN AGRICULTURAL SECTOR

Introduction

The specific-factors framework is traditionally used to analyze the short-run effects of trade liberalization. Some sectors are well characterized by the homogeneity of the produced commodity. The agricultural sector is a good example of such sector. At the same time, other sectors are better characterized by heterogeneity of firms and product differentiation. Different types of industries, such as apparel industry, are the good examples of such sectors.

Product differentiation is usually used to explain inta-industry trade among countries. It was introduced through monopolistic competition market structure in one sector model (Krugman, 1979). Melitz, 2003, introduced heterogeneous firms to the monopolistic competition market structure by Krugman, 1979 in order to account for the firm heterogeneity with respect to productivity that was found in data.

For quite a long time, tariffs were reduced substantially in manufacturing sectors but not in the agricultural sector. Given high tariffs in agricultural sector, there is a high potential for welfare improvement that would come with lowering them. Also, there is a question how such trade liberalization might effect the sectors that exhibit firm heterogeneity and product differentiation.

I am going to modify the traditional specific-factors framework by introducing the monopolistic competition market structure with heterogenous firms in one of the sectors. And then, I am going to study the effect of trade liberalization in homogeneous commodity sector on different economic indicators, such as the average productivity of firms in country's sector with differentiated commodity, the average productivity of firms there exporting abroad, and factor prices. Also, having homogeneous commodity with perfect competition market structure in the sector where trade liberalization occurs and monopolistic competition market structure with heterogenous firms in the sector affected by the spillover effect of this trade liberalization will allow for more explicit analysis of the mechanism of the spillover effect in general equilibrium framework.

Preferences and endowment structure

The analysis of trade liberalization uses a two country, two sector model in which country i has L_i endowment of labor and K_{il} endowment of sector l type capital. We begin with a description of the preferences of representative consumers and an outline of the production structure follows. We conclude with a description of the firm's entrance and exit in steady state. The words industry and sector are interchangeable.

Each country has two sectors. Sector 1 is the differentiated product sector and sector 2 is the homogeneous product sector. Many varieties of commodity are produced in sector 1, while the homogeneous commodity is produced in sector 2. The utility function of a representative consumer is:

$$U_i = \left[\left(\int_{j \in \Omega_{i1}} q_{i1}(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \right]^{\alpha} Q_{i2}^{1-\alpha}, \tag{1}$$

where $q_{i1}(j)$ denotes the consumption of variety, j, produced in industry, 1, by the representative consumer in country, i. Ω_{i1} is the set of all available varieties within industry, 1. Q_{i2} is the consumption of sector 2 commodity. α corresponds to the portion of total expenditures that goes toward the varieties in sector 1. $\sigma > 1$ restricts substitutability between varieties in sector 1. The utility of a representative consumer increases in the number of varieties and in their quantities. Taste in both countries for variety produced in the other country generates two-way trade within industry 1.

These preferences generate the following demand for variety j:

$$q_i(j) = \frac{p_i(j)^{-\sigma}}{P_i^{1-\sigma}} \alpha I_i, \tag{2}$$

where I_i is the income of a representative consumer in country, i, and $P_i = \left[\int_{j \in \Omega_{i1}} p_i(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$ is the price index (the inverse measure of the degree of competition), that in an additive way includes the prices of all varieties produced in sector, 1, which are available for consumption in country, i. Because of the continuum of varieties, changes in the price of any variety would have no effect on the price index and likewise on demand for other varieties. As such, there is no strategic interaction between firms producing different varieties.

Finally, the demand for the homogeneous good from consumers in country i is

$$Q_{i2} = \frac{(1-\alpha) I_i}{p_{i2}}, \tag{3}$$

where p_{i2} is commodity price.

Equilibrium in a differentiated product sector

As in Krugman, 1979, the assumption is that upon entering a market, a firm in sector 1 can costlessly differentiate its variety from those already existing in the market. Thus, a firm would rather produce a variety different from those already in the market, so that firm does not share the demand for this variety with another firm. Since no strategic interaction is present among firms, each firm behaves as a monopolist in setting the price for its variety domestically or abroad.

Every active firm in sector 1 uses Cobb-Douglas production function with productivity parameter, ϕ, which differs across firms. When producing quantity, q_{di}, for a domestic market and quantity, q_{xi}, for a foreign market, a firm pays the variable costs, $\frac{c_{i1}}{\phi}q_{di}$ and $\tau_1\frac{c_{i1}}{\phi}q_{xi}$, where the variable trade cost, $\tau_1 - 1$, enters in an "iceberg" form, and $c_{i1} = w_i^{\beta_1} r_{i1}^{1-\beta_1}$ is the unit cost not adjusted for efficiency. As a monopolist for variety it produces, the firm sets prices with a constant markup over marginal cost domestically and/or abroad $p_{di}(\phi) = \frac{c_{i1}}{\rho\phi}$ and $p_{xi}(\phi) = \tau_1\frac{c_{i1}}{\rho\phi}$, where $\rho = \frac{\sigma-1}{\sigma}$.

A firm collects variable profit, $\frac{R_{di}(\phi)}{\sigma}$, from domestic market and variable profit, $\frac{R_{xi}(\phi)}{\sigma}$, from foreign market, where

$$R_{di}(\phi) = \left[\frac{\rho\phi P_i}{c_{i1}}\right]^{\sigma-1}\alpha_1 I_i; \quad R_{xi}(\phi) = \left[\frac{\rho\phi P_k}{\tau_1 c_{i1}}\right]^{\sigma-1}\alpha_1 I_k. \tag{4}$$

Other things being equal, higher variable trade cost leads to lower revenue collected from the foreign market. Moreover, the revenue is proportional to the income and to the sector price index (inverse measures of competition) of the country, where the variety is sold.

In order to produce output, a firm in sector 1, pays a fixed cost, fc_{i1} which is proportional to the unit cost. In addition to this fixed cost, the firm must pay an additional fixed cost, $f_x c_{i1}$, if it exports. A firm pays fixed production cost and fixed exporting cost when serving both markets and when serving foreign market only. At the same time, by serving foreign market only, a firm does not collect positive variable profit from a domestic market, that would be collected otherwise. Therefore, the firm will choose to serve a domestic market only or to serve both markets. A firm serves foreign market in addition to the domestic market, if the variable profit from selling in a foreign market is higher than the fixed cost of exporting ($\frac{R_{xi}(\phi)}{\sigma} > f_x c_{i1}$). The resulting expression for the firm's profit is as in Melitz, 2003:

$$\pi_i(\phi) = \pi_{di}(\phi) + \max\{\pi_{xi}(\phi), 0\}, \tag{5}$$

where $\pi_{di}(\phi) = \frac{R_{di}(\phi)}{\sigma} - f c_{i1}$ and $\pi_{xi}(\phi) = \frac{R_{xi}(\phi)}{\sigma} - f_x c_{i1}$. $\pi_{di}(\phi)$ is the firm's profit when serving domestic market only. And $\pi_{xi}(\phi)$ is the increase in the profit that comes from exporting.

In steady state equilibrium, the factor prices, price indexes, incomes and the distribution of active firms over productivity remain constant over time.

An unbounded pool of identical firms have no knowledge of their future productivity before entering the market. The only information available to potential entrants about future productivity is the distribution (with distribution and density functions, $G(\phi)$ and $g(\phi)$) from which they will draw productivity after paying fixed entry cost, $f_e c_{i1}$, which is thereafter unretrievable. After the firm's productivity is realized, it remains constant over time. If the firm's productivity leads to a negative profit per period, the firm exits the market. Otherwise, after entry, the firm remains in the market and faces every period the possibility of been forced to leave the market because of external negative shock, that occurs with probability δ each period. Following Bernard, Redding, & Schott, 2007, I assume that factor intensities in entry, production and exporting are the same.

Since $R_{di}(\phi)$ and $R_{xi}(\phi)$ increase in productivity, $\pi_{di}(\phi)$ and $\pi_{xi}(\phi)$ increase in productivity as well. Since $\pi_{di}(0) = -fc_{i1}$ and $\pi_i(\phi)$ is positive for sufficiently large productivity, unique ϕ_{di} satisfying $\pi_i(\phi_{di}) = 0$. The firm with productivity above ϕ_{di} earns positive profit every period and remains in the market after entry. Contrarily, a firm with productivity below ϕ_{di} earns negative profit and exits immediately after entry. Further, ϕ_{di} will be referred to as **zero-profit productivity cutoff**.

Following Melitz, 2003, I define $\phi_{xi} = \inf\{\phi : \phi \geq \phi_{di} \text{ and } \pi_{xi}(\phi) \geq 0\}$. An active firm with productivity above ϕ_{xi} (which would be referred to as **exporting productivity cutoff**) exports. Zero-profit productivity cutoff might coincide with exporting productivity cutoff (Figure 1). In this case, all active firms within sector 1 export. This happens, when active firms with sufficiently low productivity collect negative profits when serving domestic market only, but gain a sufficiently high increase in profit from exporting resulting in the positive total profit. If $\phi_{xi} > \phi_{di}$, then firms divide into exporters and non-exporters (Figure 2). Firms with productivity above ϕ_{di}, but below ϕ_{xi}, sell in a domestic market only, while firms with productivity above ϕ_{xi} sell in both domestic and export markets. In this case, firms with low productivity do not attain the increase in profit from exporting and serve a domestic market only, while firms with high productivity receive the increase in profit from exporting and serve both markets. Further, we will concentrate on the case when firms in both countries are divided into non-exporters and exporters. Zero-profit productivity cutoff and exporting productivity cutoff are determined by conditions $R_{di}(\phi_{di}) = \sigma fc_{i1}$ and

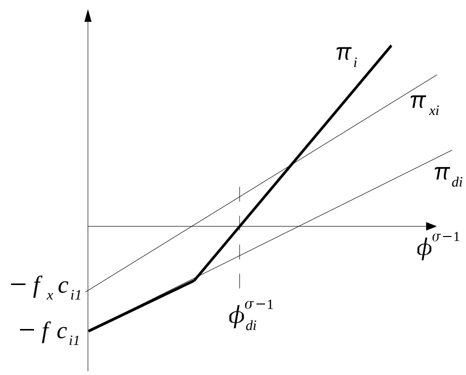

Figure 1: All firms export ($\phi_{di} = \phi_{xi}$)

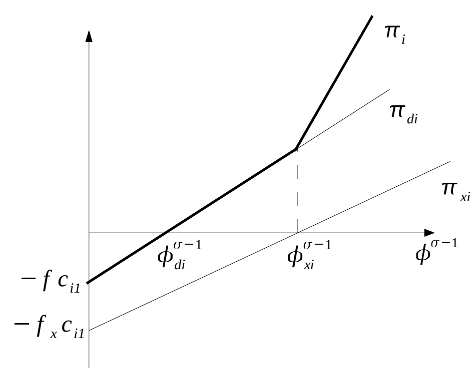

Figure 2: Firms divided into exporters and non-exporters ($\phi_{di} < \phi_{xi}$)

$R_{xi}(\phi_{xi}) = \sigma f_x c_{i1}$. Property $\frac{R_l(\phi')}{R_l(\phi'')} = \left[\frac{\phi'}{\phi''}\right]^{\sigma-1}$ in combination with the expressions for $R_{di}(\phi_{di})$ and $R_{xi}(\phi_{xi})$ leads to the expressions for the revenue of the firm with productivity ϕ on the domestic market and foreign market:

$$R_{di}(\phi) = \left[\frac{\phi}{\phi_{di}}\right]^{\sigma-1} \sigma f c_{i1}; \quad R_{xi}(\phi) = \left[\frac{\phi}{\phi_{xi}}\right]^{\sigma-1} \sigma f_x c_{i1}. \tag{6}$$

The value of entering, for a firm, would be equal to the stream of per period profits discounted by the probability of staying in the market: $V_i(\phi) = \sum_{t=0}^{\infty} (1-\delta)^t \pi_i(\phi) = \frac{\pi_i(\phi)}{\delta}$. Given the uncertainty about future productivity, the expected value of entering the market for a potential entrant would be equal to: $V_i = \frac{[1-G(\phi_{di})]}{\delta}[\overline{\pi}_{di} + \varkappa_i \overline{\pi}_{xi}]$. The potential entrant factors in the probability of making a positive per period profit, $1 - G(\phi_{di})$. The average profit includes the average profit collected from the domestic market, $\overline{\pi}_{di}$, and the average increase in profit that comes with exporting, $\overline{\pi}_{xi}$, weighted by the probability that a firm selling domestically exports, $\varkappa_i = \frac{1-G(\phi_{xi})}{1-G(\phi_{di})}$.

Since there is an unbounded pool of potential entrants, the value of entering any sector is equal to the entry cost in this sector. **Free entry condition** is:

$$\frac{[1-G(\phi_{di})]}{\delta}[\overline{\pi}_{di} + \varkappa_i \overline{\pi}_{xi}] = f_e c_{i1}. \tag{7}$$

Before entering a market, a potential entrant forms expectations for the probability of successful entrance (the probability of making positive profit) and profit, given a successful entry. The expectations are based on the information about factor prices, price indexes, distribution from which the productivity is drawn and aggregate income in every country. This information determines the zero-profit productivity cutoffs and exporting productivity cutoffs. In turn, the distribution of all active firms in any country's sector and the distribution of exporting firms in any country's sector will be determined by corresponding productivity cutoffs, since all active firms face the same exogenous probability, δ, of exiting after every period. Finally, these distributions provide the basis for finding the probability of successful entrance and the average profit, given a successful entrance.

The expressions (6) for revenues in combination with expressions for components of firm's profit $\pi_{di}(\phi)$ and $\pi_{xi}(\phi)$ lead to the following expression for free entry condition:

$$\frac{f}{\delta}\int_{\phi_{di}}^{\infty}\left[\left[\frac{\phi}{\phi_{di}}\right]^{\sigma-1}-1\right]g(\phi)\,d\phi+\frac{f_x}{\delta}\int_{\phi_{xi}}^{\infty}\left[\left[\frac{\phi}{\phi_{xi}}\right]^{\sigma-1}-1\right]g(\phi)\,d\phi=f_e. \tag{8}$$

The same intensity of factors usage in entry and production, as well as constant elasticity of demand, lead to the fact that unit cost cancels out of expression (8) corresponding to free entry condition.

The expression (8) shows the relationship between zero-profit productivity cutoff, ϕ_{di}, and exporting productivity cutoff, ϕ_{xi}, in a sector 1 of country i. The expected profit collected domestically, $(1-G(\phi_{di}))\,\overline{\pi}_{di}$, decreases with the increase in ϕ_{di}. At the same time, the increase in the expected profit from exporting, $(1-G(\phi_{xi}))\,\overline{\pi}_{xi}$, decreases with the increase in ϕ_{xi}. Since the sum of these two components should be equal to fixed entry cost, zero-profit productivity cutoff, ϕ_{di}, and exporting productivity cutoff, ϕ_{xi}, in sector 1 of country i move in opposite directions.

Here are the factors leading to the expected profit collected domestically being decreasing in ϕ_{di}. According to the expression (6) for $R_{di}(\phi)$, the increase in ϕ_{di} implies that active firm with ϕ collects smaller revenue and contributes to $\overline{\pi}_{di}$ being decreasing in ϕ_{di}. In addition, higher zero-profit productivity cutoff, ϕ_{di}, reduces the probability of successful entrance, $1-G(\phi_{di})$. This, in turn, contributes to $\overline{\pi}_{di}$ being decreasing in ϕ_{di}. At the same time, the averaging will be done over smaller interval, so $R_{di}(\phi)$ will be weighted with larger weights, $\frac{g(\phi)}{1-G(\phi_{di})}$, which contribute to $\overline{\pi}_{di}$ being increasing in ϕ_{di}. The effect of the increase in ϕ_{di} on $\frac{g(\phi)}{1-G(\phi_{di})}$ is dominated, leading to the expected profit collected domestically being decreasing in ϕ_{di}. Similar reasoning establishes that the increase in expected profit from exporting is decreasing in ϕ_{xi}.

In steady-state equilibrium, the mass of firms successfully entering a country's sector is equal to the mass of firms exiting the same sector. The following condition should hold:

$$[1-G(\phi_{di})]\, M_{ei} = \delta M_i \tag{9}$$

Equations (7) and (9) imply that the per period profit earned by active firms in sector 1 of particular country equals the entry cost paid by firms entering sector 1 of this country. As a result, the total revenue collected by firms within sector 1 of particular country is equal to the total expenditures on factors employed within sector 1 of this country. The demand for sector specific capital from firms within sector 1 should be equal to its supply K_{i1}. And, L_{i1} is the demand for labor used in production and entry created by firms in sector, 1, of country, i. Finally, it is assumed that production and trade cost parameters (f, f_x, f_e, τ_1, β_1) within sector 1 are the same across countries.

Equilibrium in a homogeneous product sector

The constant returns to scale technology is used in sector 2, with marginal cost of production to be equal to $c_{i2} = w_i^{\beta_2} r_{i1}^{1-\beta_2}$. p_{i2} is price of homogeneous commodity in country i. With constant return to scale technology, commodity price, p_{i2}, should be equal to the marginal cost of production, c_{i2}, for non-zero, finite amount of commodity being produced in equilibrium: $p_{i2} = c_{i2}$. This condition implies that the revenue collected by firms in sector 2 of country i equals to the expenditures on factors of production employed in sector 2 of this country.

In addition, the factor prices should bring the equality between the demand for sector specific capital and its exogenous supply, K_{i1}. The production of homogeneous commodity will generate the demand for labor, L_{i2}, to be employed in sector 2. Finally, the producers in sector 2 of country, exporting its output, pay the iceberg trade cost τ_2 on their exports.

Overall equilibrium

The sectors within a country are connected through labor market. The labor market clearing condition would require that the demand for labor in country i is equal to its exogenously given supply L_i:

$$L_{i1} + L_{i2} = L_i \tag{10}$$

We can establish the connection between unit costs across countries within each sector. When country 1 has comparative advantage in sector 1, it imports sector 2 commodity.

Because of the trade cost τ_2, the price of homogeneous commodity in country 1 is higher than the price of homogeneous commodity in country 2: $p_{12} = \tau_2 p_{22}$. This leads to

$$\frac{c_{12}}{c_{22}} = \tau_2. \tag{11}$$

Firms selling their output in sector 1 of country 2 face the same conditions in terms of price index, P_2, and country's income, I_2. As result, price index, P_2, and country's income, I_2, drop out from the ratio of revenues in following condition $\frac{fc_{21}}{f_x c_{11}} = \frac{R_{x2}(\phi_{x2})}{R_{d1}(\phi_{d1})}$. So that, the ratio of unit costs is proportional to the ratio of cutoffs:

$$\frac{c_{21}}{c_{11}} = \left[\frac{\phi_{d2}}{\phi_{x1}}\right]^{\frac{\sigma-1}{\sigma}} \left[\frac{f_x}{f}\right]^{\frac{1}{\sigma}} \tau_1^{\frac{\sigma-1}{\sigma}}. \tag{12}$$

In contrast to the relationship between unit costs in sector 2, the ratio of unit costs in sector 1 depends on the ratio of the productivity cutoffs as well as on trade cost parameters.

Similar condition for firms in sector 1 selling their output in the market of country, 1, can be derived. In this case, firms selling domestically in country 1 and firms exporting to country 1 face demand for their varieties driven by income I_1 and the price index, P_1, of country 1. Combining these expressions produces:

$$\frac{\phi_{x1}\phi_{x2}}{\phi_{d1}\phi_{d2}} = \tau_1^2 \left[\frac{f_x}{f}\right]^{\frac{2}{\sigma-1}}. \tag{13}$$

Since firms are subdivided into exporters and non-exporter in the type of equilibrium, we analyze, then the inequality $\tau_1 \left[\frac{f_x}{f}\right]^{\frac{1}{\sigma-1}} > 1$ should hold.

The expression (13) in combination with expression (8) written for both countries connects zero-profit productivity cutoffs and exporting productivity cutoffs within sector 1 across countries.

The income of all consumers in country i consists of the return to country's endowment of sector-specific capitals and labor, $I_i = w_i L_i + \sum_l r_{il} K_{il}$. According to conditions (7) and (9), the revenue collected by firms in sector, l, of country, i, is equal to the return to the factors of production employed in this sector, $I_{il} = w_i L_{il} + r_{il} K_{il}$. Therefore, the total return to the factors of production employed in sector, l, in both countries is equal to the expenditures on commodity produced within this sector. We have the goods market clearing condition:

$$\sum_i I_{il} = \alpha_l \sum_i I_i. \tag{14}$$

Finally, the expenditures by country, i, on goods produced within sector, 1, αI_i, become the returns to the factors of production employed by domestic and foreign firms, selling their products on country i market.

$$\alpha I_i = \gamma_i I_{i1} + [1 - \gamma_k] I_{k1}. \tag{15}$$

The part of these expenditures goes to domestic firms and becomes the return to the factors employed in sector, 1, of country, i, $\gamma_i I_{i1}$. $\gamma_i = \frac{\overline{R}_{di}}{\overline{R}_i}$ is the ratio of revenue collected domestically to the total revenue of firms within sector, 1, of country, i. I_{i1} is the revenue collected by firms in country i and sector 1, which is equal to the return to factors of production employed by these firms. The other part of these expenditures goes to foreign firms, exporting to country i. These firms collect $[1-\gamma_k] I_{k1}$. Summing the expression (15) over countries results in goods market clearing condition (14). In this sense, it is sufficient to have the relationship for expenditures of country 1 on sector 1 only and goods market clearing condition (14). The conditions outlined in this section determine the equilibrium.

Free trade

A further consideration is the trade between countries under variable-trade cost and fixed-trade cost being zero. Before exploring this case, an analysis of autarky comes first, followed by an analysis of changes in a country's economy as it transitions from autarky to free trade.

Autarky

Since in autarky, firms collect profits only on the domestic market, the free entry condition (8) transforms to

$$\frac{f}{\delta}\int_{\phi_d}^{\infty}\left[\left[\frac{\phi}{\phi_d}\right]^{\sigma-1}-1\right]g(\phi)\,d\phi=f_e. \tag{16}$$

Notice, that this condition pins down zero profit productivity cutoff, ϕ_d. With the increase in the fixed cost of production, f, ϕ_d increases. At the same time, the increase in the fixed entry cost, f_e, leads to the decrease in ϕ_d.

As demonstrated in the entry/exit part of the model specification, the income spent by consumers on products produced in sector l, $\alpha_l I$, is equal to the payment to the factors of production employed there, so that $\alpha_l I = wL_l + r_l K_l$ (goods market clearing condition). The equality, $[1-\beta_l]\, wL_l = \beta_l r_l K_l$, specifies the relationship for the expenditures on factors of production within an industry. This equality comes from the Cobb-Douglas specification of technology used in production and entry. This relationship for both industries together with goods market clearing and labor market clearing condition (10) leads to the determination of the rentals on capital as well as the allocation of labor across industries (w is normalized to unity). The Cobb-Douglas specification of technology leads to the fact, that the allocation of labor across sectors does not depend on the endowments of sector specific capital:

$$\begin{array}{c} r_l = \frac{1-\beta_l}{\beta_l}\frac{L_l}{K_l} \\ L_l = \frac{\alpha_l \beta_l}{\sum_l \alpha_l \beta_l} L \end{array} . \tag{17}$$

As the rentals on sector specific capitals, as well as zero-profit productivity cutoffs, solved, the determination of the average revenue, $R\left(\widetilde{\phi}\right)$, for every industry is possible.[1] This leads to determination of the number of active firms $M = \frac{I_1}{R\left(\widetilde{\phi}\right)}$, where $I_1 = wL_1 + r_1 K_1$ is the return to factors employed in sector 1.

The variables of interest depend on the zero-profit productivity cutoff and the rentals on sector-specific capital. In this model, the zero-profit productivity cutoff, ϕ_d, and the rentals on sector specific capital are determined by by independent set of conditions (16) and (17).

Such an independence is useful for tracing the effects of changes in different parameters on the equilibrium outcomes. We have following expression for the

[1] $\widetilde{\phi}\left(\phi_d\right)^{\sigma-1} = \frac{\int_{\phi_d}^{\infty} \phi^{\sigma-1} g(\phi) d\phi}{1-G(\phi_d)}$

price of the variety produced by firm with average productivity, $\widetilde{\phi}$:

$$p\left(\widetilde{\phi}\right)=\frac{1}{\rho\widetilde{\phi}F_1\left(L_1,K_1\right)}$$

where $F\left(L,K\right)$ is Cobb-Douglas production function[2].

The more productive firms operate within sector, 1, the lower the price set by firm with average productivity. Also, the productivity of the labor employed within sector 1 influences the price level. The higher the productivity of labor, $F_1\left(L_1,K_1\right)$, the lower prices become. Since labor is numeraire, scarce labor leads to lower relative commodity prices.

According to relationship (9), M_e is proportional to M. So, the fixed entry cost paid by entering firms is proportional to Mc_1. Since variable and fixed production costs are proportional to Mc_1, the total cost paid by firms per period is proportional to Mc_1. So, M is proportional to the output, $F\left(L_1,K_1\right)$, resulting from employment of all available factors within an sector:

$$M=\left[\frac{\widetilde{\phi}}{\phi_d}\right]^{1-\sigma}\frac{F\left(L_1,K_1\right)}{\sigma f}.$$

Since price index increases in average price and decreases in the mass of firms in the market, the increase in sector-specific capital reduces the average price as well as increases the mass of firms leading to the decrease in sector's price index. On other side, the effect of the increase in labor is not unambiguous. The mass of firms increases with labor endowment. But the average price decreases as labor becomes less productive. The first effect dominates if elasticity is sufficiently

[2] $F\left(x,y\right)=\frac{x^{\beta_1}y^{1-\beta_1}}{\beta_1^{\beta_1}\left[1-\beta_1\right]^{1-\beta_1}}$, $F_1\left(x,y\right)=\frac{\partial F(x,y)}{\partial x}$ and $F_2\left(x,y\right)=\frac{\partial F(x,y)}{\partial y}$

small:

$$P = \frac{[\sigma f]^{\frac{1}{\sigma-1}}}{\rho\phi_d} \frac{1}{[F(L_1,K_1)]^{\frac{1}{\sigma-1}} F_1(L_1,K_1)}.$$

Free trade

Under a free trade regime, both fixed cost, f_x, and variable trade costs, $\tau_1 - 1$, in sector 1 are zero. The trade cost in sector 2, $\tau_2 - 1$, is also zero in free trade. While receiving positive variable profit abroad and not paying fixed exporting cost, every firm attains an increase in profit with transition from selling domestically to selling in both markets. As result, every active firm will export: zero-profit productivity cutoff is equal to exporting productivity cutoff ($\phi_{di} = \phi_{xi}$). The fact that zero profit productivity cutoff, ϕ_{di}, and exporting productivity cutoff, ϕ_{xi}, are equal leads to the same free entry condition (16) as in the autarky case. Since cost parameters, f, f_e, and the distribution of productivity, $g(\phi)$, are assumed to be the same across countries, the zero-profit productivity cutoffs are the same across countries within a sector 1 ($\phi_{di} = \phi_{dk} = \phi_d$ and $\widetilde{\phi}_i = \widetilde{\phi}_k = \widetilde{\phi}$). Due to the fact that all active firms export and set prices domestically and abroad at the same level, we have the equality of price indexes across countries[3]. The condition for zero-profit productivity, ϕ_{di}, changes to:

$$R_i(\phi_{di}) = \left[\frac{\rho\phi_{di}P}{c_{i1}}\right]^{\sigma-1} \alpha I = \sigma f c_{i1},$$

where $I = I_i + I_k$. We can conclude that $\frac{c_{i1}}{\phi_{di}^{\frac{\sigma-1}{\sigma}}}$ is equal across countries within

[3] $P_{il} = P_{kl} = P_l = \left[M_{il} \left[\frac{w_i^{\beta_l} r_{il}^{1-\beta_l}}{\rho}\right]^{1-\sigma} \widetilde{\phi}_{il}^{\sigma-1} + M_{kl} \left[\frac{w_k^{\beta_l} r_{kl}^{1-\beta_l}}{\rho}\right]^{1-\sigma} \widetilde{\phi}_{kl}^{\sigma-1} \right]^{\frac{1}{1-\sigma}}$

sector, 1. This leads to the equality of unit costs in sector 1. According to the condition (11) , in free trade we have the equality of the unit costs in sector 2. So, for both sectors, we have

$$c_{il} = c_{kl}. \tag{18}$$

The equality of unit costs (expression (18)), the goods market clearing condition (expression (14)), the relation between the expenditure on labor and on sector-specific capital, $w_i L_{il} = \frac{\beta_l}{1-\beta_l} r_{il} K_{il}$, and labor market clearing condition (expression (10)) lead to the determination of factor prices.

Any equilibrium can be referenced by ϕ_{di} with $i = 1, 2$ and $\{w_i, r_{il}\}$ with $i, l = 1, 2$. ϕ_{il}, w_i, r_{il} lead to the determination of $R_i(\phi)$ and $\pi_i(\phi)$ as well as their average values. The allocation of labor across sectors is determined by $w_i L_{il} = \frac{\beta_l}{1-\beta_l} r_{il} K_{il}$. The mass of firms (M_i) is determine as the ratio of total revenue collected by firms within sector to the average revenue of firms in this sector. Finally, price indexes can be found from information on the mass of firms and commodity prices $p_i\left(\widetilde{\phi}_i\right)$.

Proposition 1 *A unique free trade equilibrium, referenced by $\{w_i, r_{il}, \phi_{di}\}$ with $i, l = 1, 2$ exists.*

To focus on the changes in Country 1 with transition from autarky to free trade, we normalize $w_1 = 1$. If all labor in Country i moved to sector l, then its productivity would be equal to $a_{il}^L = \beta_l \left[\frac{K_{il}}{L_i}\right]^{1-\beta_l}$. Then, $\frac{a_{i1}^L}{a_{i2}^L}$ shows how labor would be more productive in sector 1 relatively to sector 2. At the same time $\frac{a_{i1}^L}{a_{i2}^L}$ is the indicator of comparative advantage. If $\frac{a_{21}^L}{a_{22}^L} < \frac{a_{11}^L}{a_{12}^L}$, then Country 1 has a comparative advantage in sector 1, while Country 2 has a comparative advantage in sector 2.[4]

Proposition 2 *With identical factor intensities in entry, production and exporting, in transition from autarky to free trade: (a) The zero-profit productivity cutoff and average industry productivity stay the same. (b) The rental on capital relative to wage rate in the country's comparative advantage sector increases. (c) The rental on capital relative to wage rate in the country's comparative disadvantage sector decreases. (d) Labor reallocates to the country's comparative advantage sector. (e) The mass of firms increases in sector* 1*, if it is comparative advantage sector and decreases if it is comparative disadvantage sector. (f) The number of available for consumption varieties in sector* 1 *increases.*

In addition to the effect of trade liberalization within traditional approach, we have the positive effect of the increase in variety on the welfare of owners of any factor, that is specific to outlined framework. Changes in average productivity of firms might have had the effect on welfare of the owners of factors, but in transition from autarky to free trade the average productivity of firms stays the same.

Costly trade

We will start with the analysis of the modified model. In the modified framework, there are only fixed factors of production (sector specific capital) and the model does not have mobile factor (labor). The modified model is the case of

[4] When a country has a comparative advantage in some sector, it is a net exporter in this sector.

outlined framework with $\beta_1 = 0$ and $\beta_2 = 0$. We could get more explicit description of the mechanism of the effect of trade liberalization in sector 2 on the average productivity of the firms in each country within sector 1. With this modification, the unit cost includes only the cost of sector specific capital $c_{il} = r_{il}$. At the same time, the return to the sector specific capital employed in the sector l of country i is equal to $I_{il} = r_{il}K_{il}$ and the aggregate income of the residents in country i is equal to $I_i = \sum_l r_{il}K_{il}$.

This section considers positive fixed trade cost f_x and variable trade cost $\tau_1 - 1$. We are going to analyze the effect of the decrease in variable trade cost, τ_2, on economic variables. We have the following existence result for the equilibrium defined by conditions outlined in section "Overall equilibrium".

First notice that expression (8) describes the relationship between zero-profit productivity cutoff, ϕ_{di}, and exporting productivity cutoff, ϕ_{xi}, in sector 1 of country i. Similar to Bernard, Redding, & Schott, 2007, comparison of expression (16) and expression (8) leads to the conclusion that, with transition from autarky to costly trade, ϕ_{di} increases. The possibility of exporting makes market entrance more appealing and leads to the increase in the number of firms there. The increased competition between firms pushes up ϕ_{di}.

From the expression (8), the inequality for the percentage changes of the cutoffs, $\widehat{\phi}_{di}$ and $\widehat{\phi}_{xi}$, within sector 1 of country i can be derived [5]:

---

[5]

$$\nu_i = \frac{f_x \int\limits_{\phi_{xi}}^{\infty} \left[\frac{\phi}{\phi_{xi}}\right]^{\sigma-1} g(\phi)d\phi}{f \int\limits_{\phi_{di}}^{\infty} \left[\frac{\phi}{\phi_{di}}\right]^{\sigma-1} g(\phi)d\phi}$$

$$-\frac{\widehat{\phi}_{di}}{\widehat{\phi}_{xi}} = \nu_i < 1 \tag{19}$$

This implies a smaller percentage drop in $\widehat{\phi}_{di}$ in response to the percentage increase in $\widehat{\phi}_{xi}$ for the case of $f_x < f$. Fixed cost of entry equals to the sum of the expected profit collected domestically and the increase in expected profit, that comes with exporting, according to the expression (8). As result, the increase in one component should be compensated by the decrease in the other one. It could be shown that the expected profit collected domestically and the increase in expected profit, that comes with exporting, are less responsive to the changes in corresponding productivity cutoff for larger values of this cutoff. In the equilibrium of interest zero-profit productivity cutoff is smaller than exporting productivity cutoff ($\phi_{di} < \phi_{xi}$). So, the expected profit collected domestically decreases more in response to the percentage increase in $\widehat{\phi}_{di}$, than the increase in expected profit, that comes with the exporting, goes up in the response to the equivalent decrease in $\widehat{\phi}_{xi}$. For the change in the expected profit collected domestically to be equal to the the change in the increase in expected profit from exporting with opposite sign, we should have $\widehat{\phi}_{di} < -\widehat{\phi}_{xi}$.

At the same time, the condition (13) could be rewritten in percentage changes as

$$\widehat{\phi}_{x1} - \widehat{\phi}_{d1} = \widehat{\phi}_{d2} - \widehat{\phi}_{x2}. \tag{20}$$

When we choose the specific value for productivity cutoff ϕ_{d1}, the levels of all other productivity cutoffs are uniquely identified. In other words, the changes in other cutoffs could be tracked through the changes in ϕ_{d1}. Taking into account that zero-profit productivity cutoff and exporting productivity cutoff move in opposite directions, the left side of the above expression is negative, when ϕ_{d1} increases. For the right side to be negative, ϕ_{x2} needs to increase. So, ϕ_{x2} increases in the response to the increase in ϕ_{d1}.

The degree of the response of ϕ_{x2} to the increase in ϕ_{d1} determines the effect of the increase in ϕ_{d1} on the ratio of returns to capital across countries, $\frac{r_{21}}{r_{11}}$, when, instead, we consider the firms selling on country 1 market within sector 1 while deriving the expression (12). We have:

$$\frac{\widehat{r_{21}}}{r_{11}} = \frac{\sigma - 1}{\sigma} \left[\widehat{\phi}_{x2} - \widehat{\phi}_{d1} \right]. \tag{21}$$

The inequality (19) for the percentage changes in cutoff in every country and the expression (20) connecting the cutoffs across countries can be used to compare $\widehat{\phi}_{x2}$ and $\widehat{\phi}_{d1}$. The percentage decrease in ϕ_{x1}, caused by the increase in ϕ_{d1}, is larger than the percentage increase in ϕ_{d1}. At the same time, the percentage decrease in ϕ_{d2}, caused by the increase in ϕ_{x2}, is smaller than the percentage increase in ϕ_{d2}. For the left part of expression (20) to be equal to the right side, $\widehat{\phi}_{x2}$ should be larger than $\widehat{\phi}_{d1}$. So, with the increase in ϕ_{d1}, $\frac{r_{21}}{r_{11}}$ increases.

Notice that the revenue of the firms in sector l of country i, I_{il}, is equal to the return to the sector specific capital employed in this sector, $I_{il} = r_{il} K_{il}$. As result, the ratio, $\frac{I_{2l}}{I_{1l}}$, of the revenue of the firms in sector l of country, 2, to the revenue of the firms in sector l of country 1 is equal to the ratio of the returns to the sector capital across countries within sector l, $\frac{I_{2l}}{I_{1l}} = \frac{r_{2l}}{r_{1l}} \frac{K_{2l}}{K_{1l}}$. For the sector 1, since $\frac{r_{21}}{r_{11}}$ increases in ϕ_{d1}, the ratio, $\frac{I_{21}}{I_{11}}$, of the revenue of firms in country, 2, to the revenue of firms in country, 1, within sector, 1, increases in the zero-profit productivity cutoff, ϕ_{d1}.

According to the section "Overall equilibrium", the ratio of the revenue collected domestically to the total revenue collected domestically and abroad by firms within sector, 1, of country, i, is equal to $\gamma_i = \frac{\overline{R}_{di}}{\overline{R}_i}$. Where $\overline{R}_{di}$ is the average revenue collected from the domestic market and $\overline{R}_i = \overline{R}_{di} + \varkappa_i \overline{R}_{xi}$ with $\overline{R}_{xi}$ being the average revenue collected from abroad. We could rewrite γ_i as:

$$\gamma_i = \frac{\left[1 - G\left(\phi_{di}\right)\right] \overline{R}_{di}}{\left[1 - G\left(\phi_{di}\right)\right] \overline{R}_{di} + \left[1 - G\left(\phi_{xi}\right)\right] \overline{R}_{xi}}. \tag{22}$$

This representation of γ_i could be interpreted as the ratio of the expected revenue collected on the domestic market to the sum of the expected revenue collected domestically and the expected revenue collected abroad by firms within sector, 1, of country, i. Using arguments similar to the arguments in the section "Equilibrium in a differentiated product sector", we can demonstrate that the expected revenue collected domestically, $\left[1 - G\left(\phi_{di}\right)\right] \overline{R}_{di}$, decreases in zero-profit productivity cutoff, ϕ_{di}, and the expected revenue collected abroad, $\left[1 - G\left(\phi_{xi}\right)\right] \overline{R}_{xi}$, decreases in exporting productivity cutoff, ϕ_{xi}.

With the increase in the zero-profit productivity cutoff, ϕ_{di}, the exporting productivity cutoff, ϕ_{xi}, decreases. Correspondingly, the expected revenue collected on the foreign market increases and the expected revenue collected domestically decreases. As result, the share of the revenue collected domestically in the total revenue collected by firms in sector 1 of country i decreases with the increase in the zero-profit productivity cutoff, ϕ_{di}.

We have established the effect of the changes in ϕ_{d1} on the ratio, $\frac{I_{21}}{I_{11}}$, of the revenue of the firms across counties in sector 1. Also, we found the effect of the changes in ϕ_{di} on the share of domestically collected revenue in the total revenue of firms within sector 1 of each country, γ_i. Given the established properties, we can analyze the effect of the decrease in trade cost in sector 2 on the economic variables of interest. Goods market clearing condition (14) corresponding to the expenditures of both countries on sector 1 commodity ($l = 1$) and expression (15) corresponding to the expenditures of country 1 on sector 1 commodity ($i = 1$) play the important role in the analysis.

We start with the analysis of the case, when country 1 has the comparative advantage in sector 2. For normalization, we assume that $r_{12} = 1$. Before the decrease in sector 2 trade cost, τ_2, we had for the commodity prices in sector 2: $p_{22} = \tau_2 p_{12}$. When τ_2 decreases, firms in sector 2 exporting their products from country 1 to country 2 can undercut the commodity price in country 2 and make the positive profit at the same time. Facing the increased competition, firms in sector 2 and country 2 start lower their prices. Before τ_2 has been decreased, $p_{22} = r_{22}$. With the decrease in p_{22} firms in sector 2 of country 2 will be making negative profit and some of them will leave the market. As result, the demand for sector specific capital in sector 2 of country 2 decreases, leading to the decrease in r_{22} till r_{22} will be equal to the new value of τ_2.

Though the described changes occurred in sector 2, these changes will effect sector 1. With the decrease in r_{22}, the income of the owners of sector specific capital in sector 2, $I_{22} = r_{22} K_{22}$, decreases. This leads to the decrease in overall income of the residents in country 2, I_2. As result, the residents in country 2 will spend less on sector 1 commodity. Since residents in country 2 spend less on sector 1 commodity, firms exporting from country 1 to country 2 collect less revenue. The return to the sector-specific capital in sector 1 of country 1 decreases. We could note right away that with r_{12} being numeraire, $\frac{I_{11}}{I_{12}}$ decreases. Moreover, later having less income, the residents in country 1 will be spending less on sector 1 commodity leading to $\frac{I_{11}}{I_{12}}$ being smaller in resulted equilibrium than in the initial equilibrium. In other words the decrease in $\frac{I_{11}}{I_{12}}$ is the result of the decrease in τ_2 and goods market clearing condition (14)[6].

Smaller demand for sector 1 commodity from the residents in country 2 leads to more intense competition on country 2 market. The least productive firms in sector 1 of country 2, that sell domestically only, will be forced to leave market. As result, zero-profit productivity cutoff, ϕ_{d2}, in country 2 increases. And overall number of firms, M_2, based in country 2 decreases. With more intense competition, the exporting productivity cutoff, ϕ_{x1}, for the firms exporting their products from country 1 to country 2 market will increase.

Now, let's analyze the market of country 1. On the left side of the expression (15) we have the expenditures by residents of country 1 on the sector 1 commodity. And on the right side, we have the revenues of the firms toward which these expenditures went. With the assumption that I_{21} decreases in such a way that $\frac{I_{21}}{I_{11}}$ in the final equilibrium is the same as in the initial leads to the violation of the condition (15). In this case, the expenditures of country 1 on sector 1 exceed the revenue of the firms collected on this market. Notice that if the violation did not occur, than there would not exist the pressure for ϕ_{d1} (and all other cutoff) to change according to the expression (21). Since expenditures exceed revenues of the firms collected on this market, there is an opportunity for new entrants with

[6]

$\frac{I_{12}}{I_{11}}$ increases as $\frac{I_{22}}{I_{12}}$ decreases in modified goods market clearing condition: $\frac{I_{12}}{I_{11}} = \frac{1-\alpha}{\alpha} \frac{1+\frac{I_{21}}{I_{11}}}{1+\frac{I_{22}}{I_{12}}}$

realized productivity that is smaller than the productivity of the active firms on the market to enter and collect positive profit on domestic market. This will cause zero-profit productivity cutoff, ϕ_{d1}, to decrease. Moreover, because of the disbalance of expenditures and firm's revenue on this market, less productive firms from country 2 will find profitable to export their products from country 2 to country 1.

Summarizing, we have established that ϕ_{d1} decreases and ϕ_{x1} increases, while ϕ_{d2} increases and ϕ_{x2} decreases. We could see that with such changes in cutoffs, the ratio, γ_1, of the domestically collected revenue to the total revenue of the firms in country 1 increase. Also, the ratio, $1-\gamma_2$, of the revenue collected abroad to the total revenue of firms in country 2 increase. These changes lead to the increase in the revenue collected by firms selling on country 1 market (the increase in the right side of the condition (15)). At the same time the ratio, $\frac{I_{21}}{I_{11}}$, decreases with the adjustment of productivity cutoff to final equilibrium values.

We have described the mechanism of the spillover effect of the trade liberalization in sector 2 in the specific factors model without mobile factor. Similar mechanism works in the case of the specific factors model with mobile factor (labor). First, we have the following existence result for the equilibrium defined by conditions outlined in section "Overall equilibrium".

Proposition 3 *A unique costly trade equilibrium, referenced by variables:* $\{w_i, r_{il}, \phi_{di}, \phi_{xi}\}$ *with* $i, l = 1, 2$ *exists.*

The case where comparative advantage is driven by interaction between the endowments of sector specific capital, endowment labor and factor intensities across sectors (as specified in section "Free trade") is considered.

Proposition 4 *Following conditions hold:*

1. *f, f_x and f_e are assumed to be the same across countries with $f_x < f$*

2. *There exists $\overline{\beta} \geq \frac{1}{2}$ and $\beta_1 \leq \overline{\beta}$, and $\beta_2 \geq \beta_1$*

A decrease in sector 2 *trade cost leads to: (a) an increase in sector* 1 *exporting productivity cutoff (the decrease in zero-profit productivity cutoff) and a decrease in $\frac{w_2}{w_1}$, if sector* 1 *is of comparative disadvantage; (b) a decrease in sector* 1 *exporting productivity cutoff (the increase in zero-profit productivity cutoff) and an increase in $\frac{w_2}{w_1}$, if sector* 1 *is of comparative advantage; (c) an increase in the rental on capital relative to wage rate in comparative advantage sector and decrease in the rental on capital relative to wage rate in comparative disadvantage sector; and (d) labor moves from the comparative disadvantage sector to the comparative advantage sector.*

The results about the spillover effects of trade liberalization trade are unique and add value to the existing literature. Melitz, 2003, framework has heterogeneous firms with monopolistic competition market structure. At the same time, Melitz, 2003, framework is one sector model, which precludes us from exploring the spillover effects of trade liberalization in one sector on economic variable of interest in the other sector. Our framework allows for the analysis of spillover effects. Moreover, the above stated result provides the determinants of the spillover effect. From country's point of view, the sign of spillover effect of trade liberalization in sector 2 on the zero-profit productivity cutoff and exporting productivity cutoff in sector 1 depends on sector 1 being of comparative advantage or of comparative disadvantage for this country. We would like especially to point out that the model framework in Bernard, Redding, & Schott, 2007, has two sectors, the type of trade liberalization they considered is the one of the transition from autarky to costly trade. While we analyze the effect of the decrease in trade cost, while countries experience costly trade.

At the same time, the predictions about the changes in the rental on capital relative to the wage rage across sectors within country are consistent with the predictions generated by the sector specific factors model with perfect competition market structure in both sectors. This is very instructive, since we have shown that the predictions of somewhat simpler model about the changes in return on sector specific capital are still valid in more sophisticated framework with explicitly introduced firms. Here, we would like to point out that the discovered spillover effects correspond to the case when the trade liberalization occurred in the sector with perfecto competition market structure.

Conclusion

This chapter studies the effects of trade policies in the specific-factors model with homogeneous good and constant returns to scale in one sector and heterogeneity of firms and production differentiation in the other sector. The rich structure of the model allows the opportunity to analyze the effect of the reduction in trade cost as well as transition from autarky to costly trade on the lowest productivity among active firms as well as on the lowest productivity among the exporting firms in country's sector with differentiated product. The framework allows identifying how the effect of trade polices could depend on comparative advantage that is driven by interaction of the difference in the intensity of labor usage across sectors with the distribution of capital across countries and sectors as well as the distribution of labor across countries.

Falling trade costs lead to the reallocation of resources both within and across industries, changes in average productivity of firms, and changes in factor prices. The response of average productivity of firms within a country's sector with differentiated commodity to the reduction of trade cost in country's sector with homogeneous commodity is sector-dependent. The average productivity of firms in country's sector with differentiated commodity in response to the decrease in the trade cost in sector with homogeneous commodity decreases, if, in this sector country has comparative disadvantage. Or, equivalently, the zero profit productivity cutoff decrease. So that the firms with productivity lower than the lowest productivity of firms before the trade liberalization will enter and stay on the market. At the same time, the exporting productivity cutoff increases. The firms with relatively low productivities will exit market after trade liberalization in this case. Conversely, the average productivity of firms in country's sector with differentiated commodity increases, if country has comparative advantage in this sector. Naturally, the rental on capital to wage rate increases in the comparative advantage sector, and decreases in comparative disadvantage sector, and labor partially moves to comparative advantage sector.

This result leads to certain predictions about the effect of trade liberalization in agricultural sector on other sectors that produce differentiated products. This effect could be negative in a sense that less productive firms will need to exit. Or this effect could be positive in a sense that less productive firms could successfully enter the market. Undoubtedly, policy makers should take into account these spillover effects.

CHAPTER II

TRADE LIBERALIZATION WITH HETEROGENEOUS FIRMS

Introduction

In this chapter we continue to seek the answers to the questions how trade liberalization influences the distribution of firms over three dimensions: productivity, size (amounts of employed factors), and collected revenue/profit. And, we look at the spillover effects of trade liberalization in one sector on the average productivity of firms in the other sector. Though this time, we modify the traditional specific factors model by introducing monopolistic competition market structure with heterogeneous firms into both sectors. This allows us to study how the trade liberalization in the sector with heterogeneous firms and product differentiation will affect the other sector with heterogeneous firms and product differentiation. In other words, how trade liberalization in textile sector affects firms in apparel sector.

We analyze the possibility that the results, we have received in Chapter 1, might change. The reason for this is that the trade liberalization in particular sector affects the average productivity of firms in this sector. The change in average productivity of firm could affect how the trade liberalization influences the average productivity of firms in the other sector through the spillover effect. Moreover, the model setup allows us to explore the effect of trade liberalization in particular sector on the firms within the same sector in the presence of other sector. This is not possible in Melitz, 2003, since this is one sector model.

This study differs from Melitz, 2003, since we have two sectors with firms that are heterogeneous with respect to productivity. And every firm within sector produces distinct variety of sector's commodity. Having two sectors instead of one allows us to study the effect of trade liberalization in one sector on average productivity of firms in the other sector.

This study differs from Krugman, 1981, since firm are heterogeneous with respect to productivity. Having firms heterogeneous with respect to productivity leads to selfselection of firms into non-exporters and exporters. Those firms with productivities above "zero profit productivity cutoff" but below "exporting productivity cutoff" server the domestic market only. While the firms with productivities above "exporting productivity cutoff" serve both domestic market and foreign market. Trade liberalization leads to the changes in 'zero profit productivity cutoff' and "exporting productivity cutoff". Respectively, the average productivity of firms exporting abroad and at least serving domestic market changes with trade liberalization.

Further, we will outline the model setup, state the results and provide the intuition.

Preferences, endowment structure and production structure

As in Chapter I, for the analysis of trade liberalization, we use a two country, two sector model with endowment structure similar to that of the specific factors model. Country i has L_i endowment of labor and K_{il} endowment of sector l type capital.

Many varieties of commodity are produced within every sector l. So, within every sector, commodity is differentiated. The CES utility function represents preferences over a continuum (large number) of varieties within every sector. Preferences for all available varieties combine the preferences for varieties within each sector via Cobb-Douglas function, so that the share of income spent on the varieties produced within a particular sector is constant. The utility function of a representative consumer is:

$$U_i = \prod_l \left[\left(\int_{j\in\Omega_{il}} q_{il}(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \right]^{\alpha_l}, \tag{23}$$

where $q_{il}(j)$ denotes the consumption of variety, j, produced in sector, l, by the representative consumer in country, i. Ω_{il} is the set of all available to consumer varieties within industry, l. α_l corresponds to the portion of total expenditures that goes toward the varieties in industry l ($\alpha_1 = \alpha$). As result, demand for variety $q_{il}(j)$ is of the same form as in expression (2).

The production structure of sector l is similar to the production structure of sector 1 in Chapter I. Though, the fixed cost of entry, f_{el}, the fixed cost of production, f_l, the fixed cost of exporting, f_{xl}, and the variable trade cost, $\tau_l - 1$, are sector specific. The equilibrium in particular sector l is specified by conditions that are similar to the conditions in subsection "Equilibrium in a differentiated product sector" of Chapter I. The notations for zero profit productivity cutoff and exporting productivity cutoff change to ϕ_{dil} and ϕ_{xil}.

Overall equilibrium

The same labor market clearing condition (10) requires that the total demand for labor from both sectors being equal to it exogenous supply. At the same time, instead of condition (11) for the ratio of unit costs in sector 2, we have condition

$$\frac{c_{22}}{c_{12}} = \left[\frac{\phi_{d22}}{\phi_{x12}}\right]^{\frac{\sigma-1}{\sigma}} \left[\frac{f_{x2}}{f_2}\right]^{\frac{1}{\sigma}} \tau_2^{\frac{\sigma-1}{\sigma}}, \tag{24}$$

which is similar to the condition for the ratio of unit costs in sector 1, expression (12). We have conditions connecting cutoffs across countries for every sector, which is similar to the condition (13). Finally, in addition to the goods market clearing condition (14), we have two conditions stating the equivalence between the expenditures of country 1 on every sector and revenues of the firms toward which these expenditures go.

$$\alpha_l I_1 = \gamma_{1l} I_{1l} + [1 - \gamma_{2l}]\, I_{2l}. \tag{25}$$

where $l = 1, 2$. For the purposes of further analysis we can write down goods market clearing condition in the following form:

$$\frac{I_{12}}{I_{11}} = \frac{1-\alpha}{\alpha} \frac{1 + \frac{I_{21}}{I_{11}}}{1 + \frac{I_{22}}{I_{12}}}. \tag{26}$$

Welfare implications of transition from autarky to free trade

In addition to the effect of trade liberalization within traditional approach, we have the positive effect of the increase in variety on the welfare of owners of any factor, that is specific to outlined framework. Changes in average productivity of firms and in the number of firms on the market affect the welfare of the owners of factors of production. Since, in transition from autarky to free trade, the average productivity of firms stays the same, the change in the number of firms will only play role.

When the rental on capital relative to wage rate in the country's comparative advantage sector increases, the price of every variety in this sector relative to wage rate increases by smaller amount. So, the owners of capital in comparative advantage sector are able to buy larger amount of every variety produced within this sector. Also, they are be able to buy larger amount of every variety produced within comparative disadvantage sector, since the price of every variety there relative to wage rate decreases. In addition, the owners of capital have an opportunity to buy imported varieties. So they are undoubtedly better off.

At the same time, the rental on capital relative to wage rate in the country's comparative disadvantage sector decreases. The price of every variety in this sector relative to wage rate decreases by smaller amount. So, the owners of the capital in comparative disadvantage sector are able to buy the smaller amount of every variety produced within this sector. Also, they are able to buy smaller amount of every variety produced within comparative advantage sector, as price of every variety there relative to wage rate increases. In this sense, expectably they are worse off. But, the increase in the number of available varieties, which comes with trade liberalization, leads to the potential improvement in their welfare. Notice, we have following expression for welfare of the owner of capital, $U_i = \frac{r_{il}}{P_{i1}^{\alpha} P_{i2}^{1-\alpha}}$, and for price index, $P_{1l} = \frac{r_{1l}^{1-\beta_l}}{\rho \tilde{\phi}_{1l}} M_l^{\frac{1}{1-\sigma}}$. The positive effect of the increase in the number of available varieties is bigger, when σ is smaller. So, we have following result:

Proposition 5 *A value of demand elasticity $\underline{\sigma}$ exists, such that for any $\sigma < \underline{\sigma}$, the owner of any factor will gain from trade liberalization (autarky to free trade). At the same time, for sufficiently high $\sigma > \overline{\sigma}$, the owner of capital in comparative advantage sector is better off, while the owner of capital in comparative disadvantage sector is worse off in the course of transition from autarky to free trade.*

This result could be related to the similar one in Krugman, 1981. Krugman, 1981, has studied the welfare effects of trade liberalization in the two countries, two sectors model with sector specific factors and monopolistic competition market structure in both sectors. Bernard, Redding, & Schott, 2007, demonstrated that in the model with monopolistic competition market structure and heterogeneous firms under constant elasticity of demand, zero-profit productivity cutoff does not change in transition from autarky to free trade. So, basically, in the transition from autarky to free trade, we do not have productivity effect but only variety effect as in Krugman, 1981. Only in addition to specific factors, we have mobile factor (labor). When σ is sufficiently small, the variety effect dominate the decrease in the return to capital in comparative disadvantage sector. And the owners of capital in comparative disadvantage sector are better off in transition from autarky to free trade. At the same time, we would like to mention that in the two countries, two sectors factor specific model with perfect competition market structure, labor is better off from transition from autarky to free trade. (This is might not always be true in the standard specific factors model of open economy.) Since the labor is better off in the model with perfect competition market structure, we receive the improvement in welfare of labor when there exist the variety effect.

Costly trade

An idea of the effect of changes in variable trade cost on the average productivity of firms within each country's sector (or on productivity cutoffs) becomes apparent from examining how the expenditures of Country 1 on each sector are allocated across domestic and foreign firms:

$$\alpha\left[1+\frac{I_{12}}{I_{11}}\right] = \gamma_{11}+[1-\gamma_{21}]\frac{I_{21}}{I_{11}} \tag{27}$$

$$[1-\alpha]\left[1+\frac{I_{11}}{I_{12}}\right] = \gamma_{12}+[1-\gamma_{22}]\frac{I_{22}}{I_{12}}. \tag{28}$$

We can substitute the expression for $\frac{I_{12}}{I_{11}}$ from the condition (26) into the conditions (27) and (28). Then, these conditions will include the ratio of domestically collected revenue to the total revenue of firms in sector l of country i, γ_{il} with $i,l=1,2$, for both countries and both sectors and the ratio of the revenue of the firms in country 2 and the revenue of the firms in country 1 within every sector, $\frac{I_{2l}}{I_{1l}}$ with $l=1,2$.

We know that by tracking the changes in ϕ_{x1l}, the changes in all other cutoffs within sector l can be established. Moreover, we know how the changes in ϕ_{xil} influence γ_{il} and how the changes in ϕ_{x1l} influence $\frac{I_{2l}}{I_{1l}}$. By tracking only the changes in ϕ_{x1l}, we know how γ_{il}, $\frac{I_{2l}}{I_{1l}}$ and $[1-\gamma_{2l}]\frac{I_{2l}}{I_{1l}}$ will be affected. So, we could concentrate on the equilibrium values of ϕ_{x11} and ϕ_{x12} only.

Before conducting the analysis of the effect of changes in parameters on productivity cutoffs, the mechanisms determining ϕ_{x11} and ϕ_{x12} should be explored.

For equilibrium value of the ratio of wages across countries, $\frac{w_1}{w_2}$, the equilibrium values of ϕ_{x11} and ϕ_{x12} can be found from conditions (27) and (28).

With the increase in ϕ_{x11}, the right side of the expression (27) increases. The resulted decrease in $\frac{I_{21}}{I_{11}}$ causes the decrease in $\frac{I_{12}}{I_{11}}$, according to condition (26). ϕ_{x12} should increase to restore the equality in expression (27). Moreover, the increase in ϕ_{x12} should be such that the increase in $\frac{I_{21}}{I_{11}}$ resulted from the increase in ϕ_{x12} is larger than the initial decrease in $\frac{I_{21}}{I_{11}}$ resulted from the initial increase in ϕ_{x11}.

With the increase in ϕ_{x11}, the right side of the expression (28) does not change. So, the increase in $\frac{I_{21}}{I_{11}}$ resulted from the increase in ϕ_{x12} should be equal to the decrease in $\frac{I_{21}}{I_{11}}$ caused by the initial increase in ϕ_{x11}. We can conclude that the increase in ϕ_{x12} in response to the increase in ϕ_{x11} is larger for condition (27) than for condition (28). So, the curve $\phi^1_{x12}(\phi_{x11}, w)$, corresponding to condition 1, is steeper than the curve $\phi^2_{x12}(\phi_{x11}, w)$, corresponding to condition 2 (Figure 3 and Figure 4).

In this subsection, we undertake the detailed analysis of the case, when countries are equally endowed with labor. And the structure of capital endowments is:

$$\begin{array}{c} K_{11} = \lambda K_{21} \\ \lambda K_{12} = K_{22} \end{array},$$

where $\lambda > 1$. In addition, we require the total endowment of capital be the same across countries: $K_{11} + K_{12} = K_{21} + K_{22}$ and $\alpha = \frac{1}{2}$. Then country 1 has the comparative advantage in sector 1 and comparative disadvantage in sector 2.

Proposition 6 *Assuming that β_l, τ_l, f_l, f_{xl} and f_{el} are the same across sectors* *with $f_{xl} < f_l$, the decrease in sector's variable trade cost leads to: (a) a decrease in this sector's exporting productivity cutoff (and to the increase in this sector zero-profit productivity cutoff); (b) an increase in the other sector's exporting productivity cutoff (and to the decrease in zero-profit productivity cutoff in the other sector) if variable trade cost decreases in comparative advantage sector; (c) the decrease in the other sector's exporting productivity cutoff (and to the increase in zero-profit productivity cutoff in the other sector) if variable trade cost decreases in comparative disadvantage sector; (d) the increase in the rental on capital relative to wage rate in comparative advantage sector and decrease in the rental on capital relative to wage rate in comparative disadvantage sector; and (e) labor moves from the comparative disadvantage sector to the comparative advantage sector.*

This result, though derived under somewhat strong assumptions, demonstrates that we have the spillover effect of trade liberalization as stated in Chapter 1 in the modified framework with monopolistic competition market structure and heterogeneous firms. It is interesting that the properties of the spillover effect are similar to the properties of the spillover effect outlined in Chapter I, when the market structure differs across sectors. Primarily, the sign of the spillover effect depends on comparative advantage structure of the model. Bernard, Redding, & Schott, 2007, had the framework with monopolistic competition market structure and heterogeneous firms in both sectors, while assuming that the factors of production are mobile. Also, they assumed that the fixed production cost, fixed exporting cost and variable trade cost are the same across sectors. They have found many properties corresponding to the trade liberalization, when countries go from autarky to free trade. In our analysis, we analyzed the effect of the reduction in trade cost, while countries already experience costly trade. We would like to stress that this is more realistic case, since there are no so many countries that recently experienced the transition from autarky to free trade. Also, while having for the monopolistic competition market structure in both sectors, we allowing for the change in productivity in the sector where the reduction in trade costs occurs. This possibility was ruled out in Chapter I, because of perfect competition market structure.

Finally, we have established that the average productivity of firms in the sector, where trade costs decreased, increases. This result stands in agreement with Melitz, 2003, one sector framework. So, Melitz, 2003, result about the effect of trade liberalization on the same sector productivity still stays true in the model with several sectors.

We would like to start with the description of mechanism behind the effects of trade liberalization. The new values of ϕ_{x11} and ϕ_{x12} are close to those attained from analysis of expressions (27) and (28) together with the expression (14), holding $\frac{w_i}{w_k}$ fixed. And, the directions of changes in ϕ_{x11} and ϕ_{x12} are the same as the directions of changes in ϕ_{x11} and ϕ_{x12} in the case with no adjustment in the ratio wage across countries.

Let's analyze the effects of the decrease in τ_1 on economy's variable, assuming ϕ_{x11} stays fixed. The shifts of the curves $\phi^1_{x12}\left(\phi_{x11}, w\right)$ and $\phi^2_{x12}\left(\phi_{x11}, w\right)$ for fixed ϕ_{x11} resulted from the decrease in τ_1, determine new ϕ_{x11} and ϕ_{x12}. To determine these shifts, we should identify the effects of the decrease in τ_1 on the ratio, γ_{21}, of domestically collected revenue to the total revenue of firms in Country 2 and sector 1, and on ratio, $\frac{I_{21}}{I_{11}}$, of revenues of firms across countries within sector

1. Notice, that the decrease in τ_1 does not affect the ratio, γ_{11}, of domestically collected revenue to the total revenue of firms in Country 1 and sector 1, because by fixing ϕ_{x11}, ϕ_{d11} will also be fixed, according to the expression (8).

The decrease in τ_1 leads to the increase in the profit of the firms in sector 1 exporting their output from country 2 to country 1. As result, less productive firms would be able to export their output from country 2 to country 1, what causes the decrease in ϕ_{x21}. With the decrease in ϕ_{x21}, ϕ_{d21} increases according to the condition (8). The decrease in τ_1 dominates the increase in ϕ_{d21} leading to the decrease in $\frac{I_{21}}{I_{11}}$ with the decrease in τ_1. With the decrease in ϕ_{x21} and the increase in ϕ_{d21}, the ratio, γ_{21}, increases. Finally, the increase in $1-\gamma_{21}$ dominates the decrease in $\frac{I_{21}}{I_{11}}$, so that $[1-\gamma_{21}]\frac{I_{21}}{I_{11}}$ increases with the decrease in τ_1.

With the decrease in τ_1, the right side of the expression (27) increases, since $[1-\gamma_{21}]\frac{I_{21}}{I_{11}}$ increases. The resulted decrease in $\frac{I_{21}}{I_{11}}$ causes the decrease in $\frac{I_{12}}{I_{11}}$, according to goods market clearing condition (26). ϕ_{x12} should increase to restore the equality in expression (27). Moreover, the increase in ϕ_{x12} should be such that the increase in $\frac{I_{21}}{I_{11}}$ resulted from the increase in ϕ_{x12} is larger than the initial decrease in $\frac{I_{21}}{I_{11}}$ caused by the initial decrease in τ_1.

With the decrease in τ_1, the right side of the expression (28) does not change. So, the increase in $\frac{I_{21}}{I_{11}}$ resulted from the increase in ϕ_{x12} should be equal to the decrease in $\frac{I_{21}}{I_{11}}$ caused by the initial increase in ϕ_{x11}. We can conclude that the increase in ϕ_{x12} in response to the decrease in τ_1 is larger for condition (27) than for condition (28). So, the curve $\phi^1_{x12}(\phi_{x11}, w)$, corresponding to condition 1, shifts more than the curve $\phi^2_{x12}(\phi_{x11}, w)$, corresponding to condition 2. This implies the decrease in the exporting productivity cutoff in sector 1, ϕ_{x11}. As result,

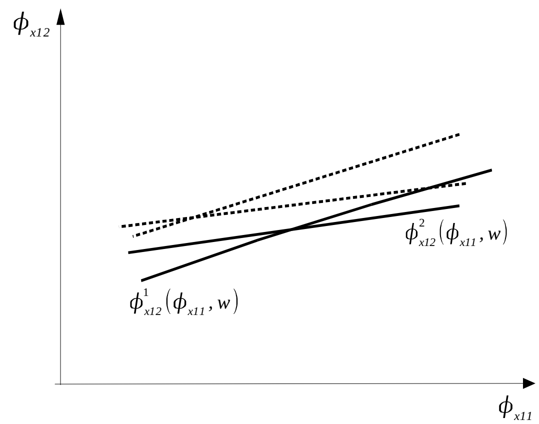

Figure 3: Change in cutoffs, when country 1 has relatively more of Sector 1 - type capital

zero-profit productivity cutoff in sector 1, ϕ_{x11}, and the average productivity of firms there increase (Figure 3 and Figure 4) in the response to the decrease in variable trade cost in the same sector.

Notice, the responsiveness of ϕ_{x12} to the increase in ϕ_{x11} is primarily determined by goods market condition (26).

$$\frac{I_{12}}{I_{11}} = \frac{1-\alpha}{\alpha} \frac{1 + \frac{r_{21}}{r_{11}} \frac{K_{21}}{K_{11}}}{1 + \frac{r_{22}}{r_{12}} \frac{K_{22}}{K_{12}}}$$

When Country 1 is has a comparative advantage in sector 1 ($\frac{K_{21}}{K_{11}}$ is smaller than $\frac{K_{22}}{K_{12}}$), then to compensate the increase in ϕ_{x11} on $\frac{I_{12}}{I_{11}}$, ϕ_{x12} should adjust by a small amount. In this case, curves are gradual (Figure 3). In the case $\frac{K_{21}}{K_{11}}$ is larger than $\frac{K_{22}}{K_{12}}$, ϕ_{x12} should adjust considerably in response to the increase in ϕ_{x11} and curves are steep (Figure 4).

Since, we have identified the shifts in curves in response to the reduction in τ_1, the analysis of how the cutoffs productivities respond to the decrease in τ_1

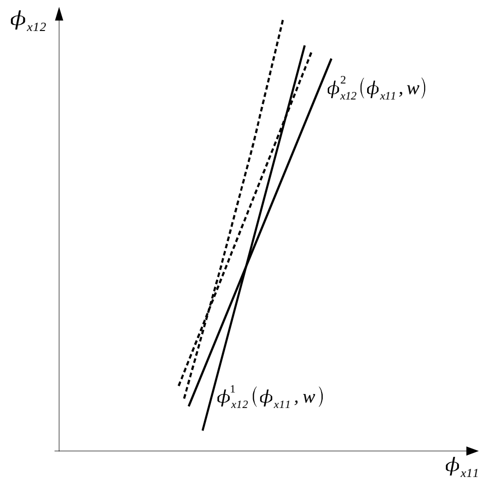

Figure 4: Change in cutoffs, when country 1 has relatively more of sector 2 - type capital

could be conducted. If the decrease in τ_1 caused the shift in curve $\phi^2_{x12}(\phi_{x11}, w)$ only, then ϕ_{x11} and ϕ_{x12} would increase (according to Figure 4 or Figure 5,). On other hand, if the decrease in τ_1 caused the shift in $\phi^1_{x12}(\phi_{x11}, w)$ only, then ϕ_{x11} and ϕ_{x12} would decrease.

The resulting effect on ϕ_{x11} is negative, because the shift in curve $\phi^1_{x12}(\phi_{x11}, w)$ is larger then the shift in curve $\phi^2_{x12}(\phi_{x11}, w)$. Only very large the shift in curve $\phi^1_{x12}(\phi_{x11}, w)$ could cause the decrease in ϕ_{x12}, when Country 1 has comparative in sector 1. So, ϕ_{x12} increases in this case (Figure 4). In this case, exporting firms with relatively low productivity will exit exporting market after trade liberalization in sector 2. Since ϕ_{d12} decreases, even the less productive firms than the ones existed on the market before trade liberalization, will be able to survive on the domestic market. At the same time, the moderate shift in curve $\phi^1_{x12}(\phi_{x11}, w)$ is needed for ϕ_{x12} to decrease, when Country 1 has comparative in sector 1. As a result, ϕ_{x12} decreases in this case (Figure 5). In this case, ϕ_{d12}

increases. The least productive firms will leave the domestic market and the firms with productivity lower, than the productivity of exporting firms before trade liberalization, will enter the exporting market successfully.

Conclusion

This chapter studies the effects of trade policies in the specific-factors model with heterogeneous firms and product differentiation. The rich structure of the model allows the opportunity to analyze the effect of the reduction in variable trade cost in particular sector on the lowest productivity among active firms as well as on the lowest productivity among the exporting firms in both country's sectors. In addition, consideration includes the effect on factor prices, price indexes, and the number of firms across sectors. The framework allows identifying how the effect of trade polices could depend on differences among sectors with respect to endowments of sector-specific capital and other sector characteristics.

Falling trade costs lead to: reallocation of resources both within and across industries, changes in average productivity of firms across sectors, and changes in factor prices. In the two-country, specific-factors model, the effect of the reduction in a sector's variable trade cost on the average productivity of firms in any one of two countries within this sector is positive.

The analysis of the case, when Country A has more capital in one sector than Country B, and Country A has less capital than country B in another sector, is conducted. In this case, the response of average productivity of firms within Country A's sector to the reduction of trade cost in the other sector is sector-dependent. The average productivity of firms in Country A's sector in response to the decrease in variable trade cost in the other sector decreases, if, in this sector, Country A has less capital than Country B. Conversely, the average productivity of firms in Country A's sector increases, if Country A has more capital in this sector than country B. Naturally, independently of the sector, where the variable trade cost decreased, the rental on capital to wage rate increases in the sector where country has more capital relative to the other country, and decreases in the other sector, and labor partially moves to the sector, where country has more capital than another country.

This result leads to certain predictions about trade liberalization, when trade costs in both sectors decrease. The average productivity of firms increases in country's sector, where the country has more capital, than the other country. In this case, the effect of the decrease in variable trade cost in other sector on average productivity of firms in this sector is positive as is the effect of the reduction in trade cost in this sector. At the same time, the average productivity of firms might increase or decrease in country's sector, where the country has less capital, than the other country. Because, the effect of the decrease in variable trade cost in other sector on average productivity of firms is negative, while the effect from the reduction in trade cost in this sector is positive. To test these result, the empirical study should be conducted.

Appendix

Proof of Proposition 1

Proof. Since any equilibrium can be referenced by $\{w_i, r_{il}, \phi_{di}\}$ with $i, l = 1, 2$, let's show that these variables are uniquely determined. ϕ_{di} can be found uniquely from condition (16), in free trade regime $\phi_{di} = \phi_d$. The left side of this expression is continuous and decreasing function of ϕ_{di}. This will guarantee the existence and the uniqueness of ϕ_{di}.

The relations between the expenditure on labor and sector specific capitals in both countries, the goods market clearing condition (14) and the equality of unit costs lead to the following system of equations (we normalized the w_1 to unity):

$$\begin{aligned} L_1 &= \tfrac{\beta_1}{1-\beta_1} r_{11} K_{11} + \tfrac{\beta_2}{1-\beta_2} r_{12} K_{12} \\ w L_2 &= \tfrac{\beta_1}{1-\beta_1} r_{21} K_{21} + \tfrac{\beta_2}{1-\beta_2} r_{22} K_{22} \\ \tfrac{1-\alpha}{1-\beta_1} \left[r_{11} K_{11} + r_{21} K_{21} \right] &= \tfrac{\alpha}{1-\beta_2} \left[r_{12} K_{12} + r_{22} K_{22} \right] \\ r_{11} &= w^{\frac{\beta_1}{1-\beta_1}} r_{21} \\ r_{12} &= w^{\frac{\beta_2}{1-\beta_2}} r_{22} \end{aligned} \tag{29}$$

The expressions for r_{11} and r_{12} from the last two equations can be substituted into first three equations. The resulting system will consist of three equations with three unknowns. Assuming that in equilibrium $K_{11} K_{22} w^{\frac{\beta_2}{\beta_2 - 1}} - K_{12} K_{21} w^{\frac{\beta_1}{\beta_1 - 1}} \neq 0$ holds (we will check this later), the first two equations can be solved for r_{21} and r_{22}. Substitute the expressions for r_{21} and r_{22} into third equation, we will get equation in w only.

$$\frac{1-\alpha}{\beta_1} \frac{w^{\frac{\beta_1}{1-\beta_1}} + \frac{1}{k_1}}{w^{\frac{1}{1-\beta_1}} - \frac{l}{k_1}} = \frac{\alpha}{\beta_2} \frac{w^{\frac{\beta_2}{1-\beta_2}} + \frac{1}{k_2}}{\frac{l}{k_2} - w^{\frac{1}{1-\beta_2}}} \tag{30}$$

where $l = \frac{L_1}{L_2}$, $k_1 = \frac{K_{11}}{K_{21}}$ and $k_2 = \frac{K_{12}}{K_{22}}$. These variables indicate the abundance of country 1 relative to country 2 with factors of production. The values of w that bring the denominator of the left and right sides to zero are $w_l = \left[\frac{l}{k_1}\right]^{1-\beta_1}$ and $w_r = \left[\frac{l}{k_2}\right]^{1-\beta_2}$ correspondingly. Since country 1 has the comparative advantage in sector 1, then $w_l < w_r$.

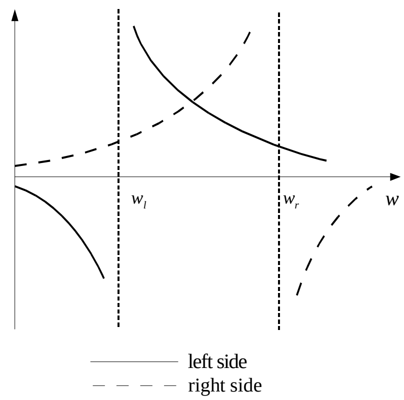

Figure 5: Determination of w

The left side of expression (30) is decreasing from infinity to some finite value on interval (w_l, w_r), while the right side of this expression is increasing from some finite value to infinity on this interval. As result, there is $w \in (w_l, w_r)$ that makes left side equal to the right side. There is no other positive value of w that satisfies equation (30).

Finally, let's check if condition $K_{11}K_{22}w^{\frac{\beta_2}{\beta_2-1}} - K_{12}K_{21}w^{\frac{\beta_1}{\beta_1-1}} \neq 0$ (or equivalently $k_1 w^{\frac{\beta_1}{1-\beta_1}} - k_2 w^{\frac{\beta_2}{1-\beta_2}} \neq 0$) is satisfied. In equilibrium we will have $w^{\frac{1}{1-\beta_1}} - \frac{l}{k_1} > 0$ and $\frac{l}{k_2} - w^{\frac{1}{1-\beta_2}} > 0$. We can write down these conditions as $k_1 w^{\frac{\beta_1}{1-\beta_1}} - \frac{l}{w} > 0$ and $\frac{l}{w} - k_2 w_2^{\frac{\beta_2}{1-\beta_2}} > 0$. So that we have $k_1 w^{\frac{\beta_1}{1-\beta_1}} - k_2 w^{\frac{\beta_2}{1-\beta_2}} > 0$. It could be shown that in this case country 1 will be net importer of sector 2 commodity. ■

Proof of Proposition 2

Proof. It was demonstrated that the zero-profit productivity cutoff and average industry productivity does not change as country moves from autarky to free trade. Let's find expressions for r_{11} and r_{12}. We can substitute the expressions for r_{21} and r_{22} from the last two equations of the system (29) into the third equation. And then, we can solve for r_{11} and r_{12} the first equation and modified third equation of that system.

$$
\begin{aligned}
r_{11} &= r_1^a \frac{\alpha\beta_1+[1-\alpha]\beta_2}{\frac{\alpha\beta_1+[1-\alpha]\beta_2}{1+\frac{1}{k_1}w^{-\frac{\beta_1}{1-\beta_1}}}\frac{}{1+\frac{1}{k_2}w^{-\frac{\beta_2}{1-\beta_2}}}} \\
r_{12} &= r_2^a \frac{\alpha\beta_1+[1-\alpha]\beta_2}{\frac{[1-\alpha]\beta_2+\alpha\beta_1}{1+\frac{1}{k_2}w^{-\frac{\beta_2}{1-\beta_2}}}\frac{}{1+\frac{1}{k_1}w^{-\frac{\beta_1}{1-\beta_1}}}}
\end{aligned}
\tag{31}
$$

From $\left[\frac{l}{k_1}\right]^{1-\beta_1} < \left[\frac{l}{k_2}\right]^{1-\beta_2}$, it follows that $k_1 w^{\frac{\beta_1}{1-\beta_1}} - k_2 w^{\frac{\beta_2}{1-\beta_2}} > 0$ (see proof of proposition 1). Or, equivalently, $\frac{1}{k_2}w^{-\frac{\beta_2}{1-\beta_2}} - \frac{1}{k_1}w^{-\frac{\beta_1}{1-\beta_1}} > 0$. From the above expressions for rentals on capital, we can see that r_{11} is increasing, while r_{12} is decreasing with transition from autarky to free trade. Since $L_{1l} = \frac{\beta_l}{1-\beta_l} r_{1l} K_{1l}$, then with transition to free trade labor will partially move to the comparative advantage sector.

Since, in free trade, every firm sells on domestic market and on foreign market, the expression for the mass of firms within country's sector would be the same as in case of autarky: $M_i = \frac{1}{1-\beta_1}\left[\frac{\phi_i}{\widetilde{\phi}_i}\right]^{\sigma-1}\frac{r_{i1}^{\beta_1}K_{i1}}{\sigma f}$. Given that the value of ϕ_i in free trade is the same as in autarky, changes in r_{i1} determine changes in M_i. As result, M_1 increases with transition to free trade, since country 1 has comparative advantage in sector. If country 1 had comparative disadvantage in sector 1, then M_1 would have decreased.

The number of available varieties in sector 1 goes up, since M_1 goes up (country 1 has comparative advantage in sector 1). Let's consider the case, when country 1 has comparative disadvantage in sector 1. The number of varieties produced in country 1, M_1, decreases in this case.

But the number of available varieties might increase because of imports of varieties from country 2. Let's check that the number of available varieties within sector 1 goes up in this case. The number of available varieties produced by firms within sector 1 in free trade regime is $M^a = M_1 + M_2 = \frac{r_{11}^{\beta_1} K_{11}}{[1-\beta_1]\sigma f} \left[\frac{\phi_1}{\widetilde{\phi}_1}\right]^{\sigma-1} \left[1 + \frac{1}{k_1 w^{\frac{\beta_1}{1-\beta_1}}}\right]$. This result follows from the fact that zero-profit productivity cutoffs and unit costs within industry are the same across countries. So, we will have

$$\frac{M_1}{M^a} = \left[\frac{\alpha\beta_1}{\alpha\beta_1+[1-\alpha]\beta_2} \frac{1+\frac{1}{k_2} w^{-\frac{\beta_2}{1-\beta_2}}}{1+\frac{1}{k_1} w^{-\frac{\beta_1}{1-\beta_1}}} + \frac{[1-\alpha]\beta_2}{\alpha\beta_1+[1-\alpha]\beta_2}\right]^{\beta_1} \frac{1}{\left[1+\frac{1}{k_2} w^{-\frac{\beta_2}{1-\beta_2}}\right]^{\beta_1}} \frac{1}{\left[1+\frac{1}{k_1 w^{\frac{\beta_1}{1-\beta_1}}}\right]^{1-\beta_1}}$$

As all of the factors are greater than unity, the number of varieties in sector 1, when country 1 has comparative disadvantage in sector 1, increases. So, the number of available varieties in sector 1 increases independently if this is the sector of comparative advantage or disadvantage. ∎

Relation between cutoffs within sector

We have two equations (8) corresponding to different countries but the same sector 1. In addition, we have equation (13), that connects all cutoffs within sector 1. Finally, we have

$$\begin{gathered}
\frac{f}{\delta}\int\limits_{\phi_{d1}}^{\infty}\left[\left[\frac{\phi}{\phi_{d1}}\right]^{\sigma-1}-1\right]g(\phi)\,d\phi+\frac{f_x}{\delta}\int\limits_{\phi_{x1}}^{\infty}\left[\left[\frac{\phi}{\phi_{x1}}\right]^{\sigma-1}-1\right]g(\phi)\,d\phi=f_e \\
\frac{f}{\delta}\int\limits_{\phi_{d2}}^{\infty}\left[\left[\frac{\phi}{\phi_{d2}}\right]^{\sigma-1}-1\right]g(\phi)\,d\phi+\frac{f_x}{\delta}\int\limits_{\phi_{x2}}^{\infty}\left[\left[\frac{\phi}{\phi_{x2}}\right]^{\sigma-1}-1\right]g(\phi)\,d\phi=f_e \\
\frac{\phi_{d1}\phi_{d2}}{\phi_{x1}\phi_{x2}}=\frac{1}{\tau_1^2}\left[\frac{f}{f_x}\right]^{\frac{2}{\sigma-1}}
\end{gathered} \tag{32}$$

These equations connect cutoffs within sector 1. As a system consists of three equations and contains four unknown variables, we could idnetify three of four variables. For this purpose, we will use ϕ_{x1} as parameter. First equation defines relationship $\phi_{d1}(\phi_{x1})$. $\phi_{d1}(\phi_{x1})$ is decreasing and convex function in positive ortant with $\underset{\phi_{d1}\to 0}{lim}\ \phi_{x1}=\infty$ and $\underset{\phi_{x1}\to 0}{lim}\ \phi_{d1}=\infty$. As result, for given ϕ_{x1}, there is always unique value of ϕ_{d1}. Then, given ϕ_{x1} and ϕ_{d1}, from third equation we could find the ratio $\frac{\phi_{d2}}{\phi_{x2}}=a$. Finally, we could guarantee that increasing linear function $\phi_{d2}=a\phi_{x2}$ and decreasing function $\phi_{d2}(\phi_{x2})$ corresponding to the second equation, with $\underset{\phi_{d2}\to 0}{lim}\ \phi_{x2}=\infty$ and $\underset{\phi_{x2}\to 0}{lim}\ \phi_{d2}=\infty$, intersect at one point.

The range for ϕ_{x1} to guaranty existence of exporters and non-exporters in both countries

From the third equation of the system, we can conclude that the necessary condition for existence of exporters and non-exporters in both countries ($\frac{\phi_{d1}}{\phi_{x1}}<1$ and $\frac{\phi_{d2}}{\phi_{x2}}<1$) is $\frac{1}{\tau_1^2}\left[\frac{f}{f_x}\right]^{\frac{2}{\sigma-1}}<1$. Assuming that this property holds, let's show that for some range of ϕ_{x1} there firms are divided into exporters and non-exporters in both countries within sector 1. As, $\phi_{d1}(\phi_{x1})$ is decreasing function, then $\frac{\phi_{d1}}{\phi_{x1}}(\phi_{x1})$ would also be the decreasing function with $\frac{\phi_{d1}}{\phi_{x1}}(0)=\infty$ and

$\frac{\phi_{d1}}{\phi_{x1}}(\infty) = 0$. So, there exists ϕ_{x1} such that $\frac{\phi_{d1}}{\phi_{x1}}(\phi_{x1}) = 1$. This is the case, when barely there are non-exporters in the first country. This condition and $\frac{1}{\tau_1^2}\left[\frac{f}{f_x}\right]^{\frac{2}{\sigma-1}} < 1$ imply existence of exporters and non-exporters in country 2, as $\frac{\phi_{d2}}{\phi_{x2}} < 1$ from the third equation. Together condition $\frac{\phi_{d1}}{\phi_{x1}} = 1$ and first equation determine $\underline{\phi}_{x1}$ (the lowest ϕ_{x1}, when there non-exporters and exporters in both countries). As the ratio $\frac{\phi_{d1}}{\phi_{x1}}$ continues to decrease with increase in ϕ_{x1}, it would become equal to $\frac{1}{\tau_1^2}\left[\frac{f}{f_x}\right]^{\frac{2}{\sigma-1}} < 1$. This is the case, when barely there are non-exporters in country 2, as $\frac{\phi_{d2}}{\phi_{x2}} = 1$. Again, the first equation together with condition $\frac{\phi_{d1}}{\phi_{x1}} = \frac{1}{\tau_1^2}\left[\frac{f}{f_x}\right]^{\frac{2}{\sigma-1}}$ can be solved for $\overline{\phi}_{x1}$. Finally, for any value of $\phi_{x1} \in \left[\underline{\phi}_{x1}, \overline{\phi}_{x1}\right]$, firms are divided into exporters and non-exporters within sector 1 in both countries.

Connection between M_i and cutoffs (when there is no mobile factor)

In the case, when there is no mobile factor in the model, we have following expression for M_i:

$$M_i = \frac{r_{il}^{\beta_l} K_{il}}{[1-\beta_l] w_i^{\beta_l} \sigma} \frac{1-G(\phi_{di})}{f \int\limits_{\phi_{di}}^{\infty} \left[\frac{\phi}{\phi_{di}}\right]^{\sigma-1} g(\phi) d\phi + f_x \int\limits_{\phi_{xi}}^{\infty} \left[\frac{\phi}{\phi_{xi}}\right]^{\sigma-1} g(\phi) d\phi}$$

Let's find the complete differential of this expression:

$$\frac{[1-\beta_l] w_i^{\beta_l} \sigma}{r_{il}^{\beta_l} K_{il}} dM_i = \frac{1}{G}\left[\left[\frac{f}{G} - 1\right] g(\phi_{di})\, d\phi_{di} + \frac{f_x g(\phi_{xi}) d\phi_{xi}}{G}\right]$$

where $G = f \int\limits_{\phi_{di}}^{\infty} \left[\frac{\phi}{\phi_{di}}\right]^{\sigma-1} g(\phi)\, d\phi + f_x \int\limits_{\phi_{xi}}^{\infty} \left[\frac{\phi}{\phi_{xi}}\right]^{\sigma-1} g(\phi)\, d\phi$. Notice that $\frac{f}{G} < 1$. So, M_i decreases with increase in ϕ_{di} (ϕ_{xi} decreases in this case)

Connection between factor prices.

Similar to free trade case, the relations between the expenditure on labor and sector specific capitals in both countries, the goods market clearing condition (14) and the relations between sector unit costs across countries (12) can be combined to the system of equations (we normalized the w_1 to unity), in which first three equations would be exactly like in the system (29). While last two conditions will modify to

$$\begin{gathered} \left[\frac{\phi_{d2}}{\phi_{x1}}\left[\frac{f_x}{f}\right]^{\frac{1}{\sigma-1}}\tau_1\right]^{\frac{\sigma-1}{\sigma[1-\beta_1]}} r_{11} = w^{\frac{\beta_1}{1-\beta_1}} r_{21} \\ \tau_2^{-\frac{1}{1-\beta_2}} r_{12} = w^{\frac{\beta_2}{1-\beta_2}} r_{22} \end{gathered} \tag{33}$$

As, $\frac{\phi_{d2}}{\phi_{x1}}$ is the function of ϕ_{x1}, then ϕ_{x1} and factor prices are the unknown variables, that enter this system. Notice that the ratio of unit costs across countries depends on values of cutoffs. In the way, we went from the system (29) for factor prices to the one equation in w, for costly trade case, we could go to the equation in ϕ_{x1} and w. We get

$$\frac{1-\alpha}{\beta_1}\frac{1+A(\phi_{x1})w^{-\frac{\beta_1}{1-\beta_1}}}{\frac{w}{l}-A(\phi_{x1})w^{-\frac{\beta_1}{1-\beta_1}}} = \frac{\alpha}{\beta_2}\frac{1+Bw^{-\frac{\beta_2}{1-\beta_2}}}{Bw^{-\frac{\beta_2}{1-\beta_2}}-\frac{w}{l}} \tag{34}$$

where $A(\phi_{x1}) = \left[\frac{\phi_{d2}}{\phi_{x1}}(\phi_{x1})\left[\frac{f_x}{f}\right]^{\frac{1}{\sigma-1}}\tau_1\right]^{\frac{\sigma-1}{\sigma[1-\beta_1]}}\frac{1}{k_1}$ and $B = \tau_2^{-\frac{1}{1-\beta_2}}\frac{1}{k_2}$. The left side of (34) is increasing in $A(\phi_{x1})$ and the right side of (34) is decreasing in $\tau_2^{-\frac{1}{1-\beta_2}}$. Again, variables on the left side correspond to the sector 1 and variables on the right side corresponds to the sector 2. The values of w that bring the denominator of the left and right sides to zero are $w_l(\phi_{x1}) = [A_1(\phi_{x1})\, l]^{1-\beta_1}$ and $w_r = [Bl]^{1-\beta_2}$. Since country 1 has the comparative advantage in sector 1, we have $w_l(\phi_{x1}) < w_r$. So, the behavior of left and right sides of this equation and the determination of w is similar to ones for equation (30) (figure: Equation (30)).

Finally, the inequality below (that comes from the fact $w_l(\phi_{x1}) < w < w_r$) is necessary for valid transition from system (33) to equation (34) (similarly to free trade case).

$$A(\phi_{x1})\, w^{-\frac{\beta_1}{1-\beta_1}} < \frac{w}{l} < B w^{-\frac{\beta_2}{1-\beta_2}} \tag{35}$$

Similar to (31), we have following expressions for rentals on capital

$$\begin{aligned} r_{11} &= r_1^a \frac{\alpha\beta_1+[1-\alpha]\beta_2}{\alpha\beta_1+[1-\alpha]\beta_2 \frac{1+A(\phi_{x1})w^{-\frac{\beta_1}{1-\beta_1}}}{1+Bw^{-\frac{\beta_2}{1-\beta_2}}}} \\ r_{12} &= r_2^a \frac{\alpha\beta_1+[1-\alpha]\beta_2}{[1-\alpha]\beta_2+\alpha\beta_1 \frac{1+Bw^{-\frac{\beta_2}{1-\beta_2}}}{1+A(\phi_{x1})w^{-\frac{\beta_1}{1-\beta_1}}}} \end{aligned} \tag{36}$$

Relation between country's and industry's incomes

Conditions (27) and (26) lead

$$\alpha + [1-\alpha] \frac{1+\frac{I_{21}}{I_{11}}}{1+\frac{I_{22}}{I_{12}}} = \gamma_1 + [1-\gamma_2] \frac{I_{21}}{I_{11}}.$$

where $\gamma_i(\phi_{xi}) = \frac{1}{1+\frac{f_x\Phi(\phi_{xi})}{f\Phi(\phi_{di})}}$ with $\Phi(x) = \int_x^\infty \left[\frac{\phi}{x}\right]^{\sigma-1} g(\phi)\, d\phi$. Using the expressions for $\frac{I_{2l}}{I_{1l}}$ with $l = 1, 2$, we get

$$\alpha + [1-\alpha] \frac{1+A(\phi_{x1})w^{-\frac{\beta_1}{1-\beta_1}}}{1+Bw^{-\frac{\beta_2}{1-\beta_2}}} = \gamma_1 + [1-\gamma_2]\, A(\phi_{x1})\, w^{-\frac{\beta_1}{1-\beta_1}}. \tag{37}$$

Conditions (34) and (37) specify the relationships between w and ϕ_{x1}. Jointly, they determine the equilibrium values of w and ϕ_{x1}. Let's look closely at expression (37).

Some preliminary comparative statics (properties)

1. $\frac{\partial A(\phi_{x1})}{\partial \phi_{x1}}$ **is negative.** First $\widehat{A}(\phi_{d2},\ \phi_{x1}) = \frac{\sigma-1}{\sigma[1-\beta_1]}\left[\widehat{\phi}_{d2} - \ \widehat{\phi}_{x1}\right]$. Then given the expression (20), we have

$$\frac{\widehat{\phi}_{d2}}{\widehat{\phi}_{x1}} = \frac{1-\frac{\widehat{\phi}_{d1}}{\widehat{\phi}_{x1}}}{1-\frac{\widehat{\phi}_{x2}}{\widehat{\phi}_{d2}}}.$$

According to inequality (19), we have

$$\frac{\widehat{\phi}_{d2}}{\widehat{\phi}_{x1}} = \frac{1+\nu_1}{1+\frac{1}{\nu_2}} < 1.$$

We can conclude that $\frac{\partial A(\phi_{x1})}{\partial \phi_{x1}}$ is negative

2. $\frac{\partial[1-\gamma_2]A(\phi_{x1})}{\partial \phi_{x1}}$ **is positive for small enough** β_1. First, let's introduce notations $\lambda(\phi_{d2}, \phi_{x2}) = 1 - \gamma_2(\phi_{d2}, \phi_{x2})$ and $y = \lambda(\phi_{d2}, \phi_{x2}) A(\phi_{d2},\ \phi_{x1})$. Then we have

$$\widehat{y} = \tfrac{\sigma-1}{\sigma[1-\beta_1]}\left[\widehat{\phi}_{d2} - \ \widehat{\phi}_{x1}\right] + [\sigma-1]\gamma_2\left[\widehat{\phi}_{d2} - \widehat{\phi}_{x2}\right] + \frac{f\phi_{d2}g(\phi_{d2})\widehat{\phi}_{d2} - f_x\phi_{d2}g(\phi_{x2})\frac{1}{\nu_2}\widehat{\phi}_{x2}}{f\int_{\phi_{d2}}^{\infty}\left[\frac{\phi}{\phi_{d2}}\right]^{\sigma-1}g(\phi)d\phi}\gamma_2$$

Equivalently, we have

$$\widehat{y} = [\sigma-1]\left[\tfrac{1}{\sigma[1-\beta_1]}\ \tfrac{\nu_1}{1+\nu_1} + \tfrac{1}{\nu_2}\left[1 - \tfrac{1}{\sigma[1-\beta_1]}\ \tfrac{1}{1+\nu_1}\right]\right]\widehat{\phi}_{d2} + \frac{f\phi_{d2}g(\phi_{d2})\widehat{\phi}_{d2} - f_x\phi_{d2}g(\phi_{x2})\frac{1}{\nu_2}\widehat{\phi}_{x2}}{f\int_{\phi_{d2}}^{\infty}\left[\frac{\phi}{\phi_{d2}}\right]^{\sigma-1}g(\phi)d\phi}\gamma_2$$

If $\sigma[1-\beta_1] > 1$, then $\widehat{y} > 0$. So, we can conclude that $\frac{\partial[1-\gamma_2]A(\phi_{x1})}{\partial \phi_{x1}} > 0$.

3. $\frac{\partial A(\phi_{x1l})}{\partial \tau_l}$ **is positive** We need to find the derivative of $\frac{\phi_{d2l}}{\phi_{x1l}}(\phi_{x1l})\tau_l$ with respect to τ_l. As ϕ_{x1l} stays fixed (partial derivative with respect to τ_l), we only need to check the behavior of $\tau_l \phi_{d2l}$. From the third equation of the system 13, we will get $\phi_{x2l} = \phi_{d2l}\tau_l^2 \left[\frac{f_{xl}}{f_l}\right]^{\frac{2}{\sigma-1}} \frac{\phi_{d1l}}{\phi_{x1l}}$. Now, substituting this expression into second equation of the system 32, we will have

$$f_l \int_{y\frac{1}{\tau_l}}^{\infty} \left[\left[\frac{\phi}{y\frac{1}{\tau_l}}\right]^{\sigma-1} - 1\right] g(\phi)\, d\phi + f_{xl} \int_{z}^{\infty} \left[\left[\frac{\phi}{z}\right]^{\sigma-1} - 1\right] g(\phi)\, d\phi = \delta f_e$$

where $y = \tau_l \phi_{d2l}$ and $z = y\tau \left[\frac{f_x}{f}\right]^{\frac{2}{\sigma-1}} \frac{\phi_{d1}}{\phi_{x1}}$. After differentiation, we have

$$\frac{d(\tau_l \phi_{d2l})}{\tau_l \phi_{d2l}} = \frac{f_l \int\limits_{\phi_{d2l}}^{\infty} \left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1} g(\phi) d\phi - f_{xl} \int\limits_{\phi_{x2l}}^{\infty} \left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1} g(\phi) d\phi}{f_l \int\limits_{\phi_{d2l}}^{\infty} \left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1} g(\phi) d\phi + f_{xl} \int\limits_{\phi_{x2l}}^{\infty} \left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1} g(\phi) d\phi} \frac{d\tau_l}{\tau_l} \tag{38}$$

As $f_{xl} < f_l$ and $\phi_{d2l} < \phi_{x2l}$, $\tau_l \phi_{d2l}$ increases with τ_l.

4. $\frac{\partial[1-\gamma_{2l}]}{\partial \tau_l}$ **is negative and** $\frac{\partial[1-\gamma_{2l}]A(\phi_{x1l})}{\partial \tau_l}$ **is negative for small enough** β_l Basically, we will need to find the derivative with respect to of

$$\frac{1}{1+\frac{f_l \int_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1} g(\phi) d\phi}{f_{xl} \int_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1} g(\phi) d\phi}} \left[\phi_{d2l}\tau_l\right]^{\frac{\sigma-1}{\sigma[1-\beta_l]}}$$

We already have expression for derivative $\frac{d(\tau_l \phi_{d2l})}{d\tau_l}$, so we need to find the derivative of the first multiplier. First, let's find $\frac{d\phi_{d2l}}{d\tau_l}$ and $\frac{d\phi_{x2l}}{d\tau_l}$. By taking the complete differential of the second and the third equations of the system

32 for the case of fixed ϕ_{x1l} (ϕ_{d1l} in this case is fixed too), we could find expressions for $\frac{d\phi_{d2l}}{d\tau_l}$ and $\frac{d\phi_{x2l}}{d\tau_l}$. Now, substituting these expressions into the formula for the complete differential of the first multiplier, we have

$$\frac{d\left[\frac{1}{1+\frac{f_l\int_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi}{f_{xl}\int_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi}}\right]}{d\tau_l}=$$

$$=-\frac{2}{\tau_l}\frac{[\sigma-1]f_{xl}\int_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi f_l\int_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi}{\left[f_l\int_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi+f_{xl}\int_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi\right]^2}-$$

$$-\frac{2}{\tau_l}\frac{f_{xl}f_l\left[f_{xl}\left[\int\limits_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi\right]^2g(\phi_{d2l})\phi_{d2l}+f_l\left[\int\limits_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi\right]^2g(\phi_{x2l})\phi_{x2l}\right]}{\left[f_l\int_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi+f_{xl}\int_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi\right]^3}$$

At this point, we can conclude that $\frac{\partial[1-\gamma_{2l}]}{\partial\tau_l}$ has negative sign. Finally, we have

$$\frac{\tau_l}{[\phi_{d2l}\tau_l]^{\frac{\sigma-1}{\sigma[1-\beta_l]}}}\frac{d\left[\frac{1}{1+\frac{f_l\int_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi}{f_{xl}\int_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi}}[\phi_{d2l}\tau_l]^{\frac{\sigma-1}{\sigma[1-\beta_l]}}\right]}{d\tau_l}=$$

$$=\left[-2+\frac{1}{\sigma[1-\beta_l]}\right]\frac{[\sigma-1]f_{xl}\int\limits_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi f_l\int\limits_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi}{\left[f_l\int\limits_{\phi_{d2l}}^{\infty}\left[\frac{\phi}{\phi_{d2l}}\right]^{\sigma-1}g(\phi)d\phi+f_{xl}\int\limits_{\phi_{x2l}}^{\infty}\left[\frac{\phi}{\phi_{x2l}}\right]^{\sigma-1}g(\phi)d\phi\right]^2}-\Lambda$$

where Λ is positive expression. We could guarantee that the expression above is negative if following condition holds $\beta_l<1-\frac{1}{2\sigma}$. So, for small enough β_l, $\frac{\partial[1-\gamma_{2l}]A(\phi_{x1l})}{\partial\tau_l}$ is negative.

5. $\phi^2_{x1}(w)$**, defined by the expression** (37) **is increasing function**.

The right side of expression (37) is decreasing in w. Let's take the derivative of the left side with respect to w. Let's denote $A = A(\phi_{x1}) \frac{1}{k_1}$ and $B = \tau_2^{-\frac{1}{1-\beta_1}} \frac{1}{k_2}$. Then

$$\frac{1+Aw^{-\frac{\beta_1}{1-\beta_1}}}{1+Bw^{-\frac{\beta_2}{1-\beta_2}}} \to \frac{\frac{1}{w}\left[\frac{\beta_2}{1-\beta_2}Bw^{-\frac{\beta_2}{1-\beta_2}} - \frac{\beta_1}{1-\beta_1}Aw^{-\frac{\beta_1}{1-\beta_1}}\right] + \left[\frac{\beta_2}{1-\beta_2} - \frac{\beta_1}{1-\beta_1}\right] ABw^{-\frac{1-\beta_1\beta_2}{[1-\beta_2][1-\beta_1]}}}{\left[1+Bw^{-\frac{\beta_2}{1-\beta_2}}\right]^2}$$

If $\beta_2 \geq \beta_1$, then given inequality (35), the left side of expression (37) is increasing in w. To bring the equality in expression (37), when ϕ_{x1} increased, w should increase. So, the relationship $\phi^2_{x1}(w)$, defined by the expression (37), is positive.

Proof of Proposition 3 and of Proposition 4

Proof. We substitute out $1 + Bw^{-\frac{\beta_2}{1-\beta_2}}$ from expressions (34) and (37), we have

$$\begin{aligned} 0 = [1-\alpha]\left[1 + Aw^{-\frac{\beta_1}{1-\beta_1}}\right] - \alpha\frac{\beta_1}{\beta_2}\left[\frac{w}{l} - Aw^{-\frac{\beta_1}{1-\beta_1}}\right] - \\ - \left[1 + \frac{w}{l}\right]\left[\gamma_1 - \alpha + [1-\gamma_2] Aw^{-\frac{\beta_1}{1-\beta_1}}\right] \end{aligned} \tag{39}$$

Notice, the right side of this expression is decreasing in ϕ_{x1}. Let's take the derivative of the right side with respect to w, we have

$$\left[\alpha - \gamma_1 - \alpha\frac{\beta_1}{\beta_2}\right]\frac{1}{l} + \frac{\beta_1}{1-\beta_1}\left[\alpha - \gamma_2 - \alpha\frac{\beta_1}{\beta_2}\right] Aw^{-\frac{1}{1-\beta_1}} + \frac{2\beta_1 - 1}{1-\beta_1}\frac{1}{l}[1-\gamma_2] Aw^{-\frac{\beta_1}{1-\beta_1}}$$

We could guarantee that this expression is negative if $\alpha - \gamma_1 - \alpha\frac{\beta_1}{\beta_2} < 0$, $[1-\gamma_2] - [1-\alpha] - \alpha\frac{\beta_1}{\beta_2} < 0$ and $\beta_1 \leq \frac{1}{2}$. Recall that $1 > \gamma_i > \frac{1}{2}$. Then, the

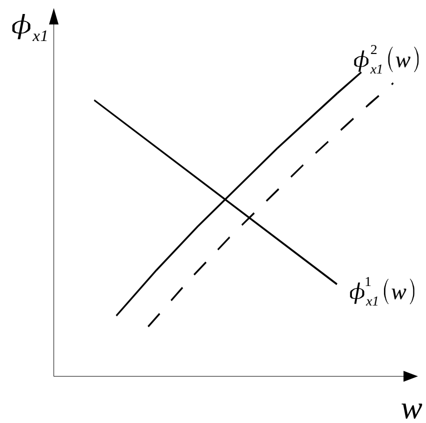

Figure 6: Changes in ϕ_{x1} and w, when τ_2 decreases

stronger versions of first two inequalities are implied by $\alpha - \alpha\frac{\beta_1}{\beta_2} \leq \frac{1}{2}$. So, the expression on the right side is decreasing in w and ϕ_{x1}. That means that the locus $\phi^1_{x1}(w)$ corresponding to the expression (39) is decreasing function.

Also, we established that the locus $\phi^2_{x1}(w)$ corresponding to the expression (37) is increasing function. The crossing of these curves corresponds to the equilibrium.

Notice, that the decrease in τ_2 leads to the vertical downward shift of the curve $\phi^2_{x1}(w)$ in case country 1 has the comparative disadvantage in sector 2. So, the decrease in τ_2 causes the increase in w and the decrease in ϕ_{x1}. It implies the increase in ϕ_{d1} or the increase in average productivity of firms in sector 1. In the same way, it could be shown that ϕ_{x1} increases in response to the decrease in τ_2, if country 1 has the comparative advantage in sector 2 and comparative disadvantage in sector 1. ■

Proof of Proposition 5

Proof. After substituting the expression (2) for demand into the expression

(23) for utility, we will get that the utility level of agent with the income I_i is $U_i = \alpha^\alpha [1-\alpha]^{[1-\alpha]} \frac{I_i}{P_{il}^\alpha P_{il}^{1-\alpha}}$. So, the changes in welfare of the owner of one unit of capital in sector l of country 1 are determined by following ratio:

$$\frac{U_{1l}^a}{U_{1l}^f} = \frac{r_{1l}^a}{r_{1l}} \left[\frac{P_{11}}{P_{11}^a}\right]^\alpha \left[\frac{P_{12}}{P_{12}^a}\right]^{1-\alpha}$$

where U_{1l}^f is utility level of the owner of one unit of capital in sector l of country 1 in free trade regime. Increase in welfare corresponds to $\frac{U_{1l}^a}{U_{1l}^f} < 1$. For both industries we have the ratios of price indices in the expression below. At the same time, rental on capital goes in sector 1 ($\frac{r_{11}^a}{r_{11}} > 1$). So, by showing that in $\frac{U_{11}^a}{U_{11}^f} < 1$, we will have automatically $\frac{U_{1l}^a}{U_{1l}^f} < 1$.

The ratio of price indices, corresponding to different trade regimes, can be expressed as

$$\frac{P_{il}^f}{P_{il}^a} = \left[\frac{r_{il}}{r_{il}^a}\right]^{1-\beta_l} \left[\frac{M_{il}^a}{M_l}\right]^{\frac{1}{\sigma-1}}.$$

Both ratios $\frac{r_{il}}{r_{il}^a}$ and $\frac{M_{il}}{M_l}$ do not depend on σ and only determined by factor endowments of the economy and $\{\alpha_l, \beta_l\}$ for $l = 1, 2$. Moreover, we have showed in the proof of proposition 2 that $\frac{M_{il}^a}{M_l} < 1$ for both industries. So, by making σ close enough to unity, we make $\frac{P_{il}^f}{P_{il}^a}$ being as small as it is necessary. As result, we can ensure that $\frac{U_{1l}^a}{U_{1l}^f}$ would be smaller than unity.

For the owner of the unit of labor, we have following ratio of utilities

$$\frac{U_{1l}^a}{U_{1l}^f} = \left[\frac{P_{11}}{P_{11}^a}\right]^\alpha \left[\frac{P_{12}}{P_{12}^a}\right]^{1-\alpha}$$

Again, by choosing σ close enough to unity, this ratio will be smaller than unity. So, there is $\overline{\sigma}$ such that for any $\sigma < \overline{\sigma}$, the owner of any factor will be better off from trade liberalization (autarky to free trade).

At the same time, substituting the expression for the ratio of price indices into the expression for the ratio of utility levels of the owner of unit of capital in sector 1 and 2, we get

$$\begin{aligned}\frac{U_{11}^a}{U_{11}^f} &= \left[\frac{r_{11}^a}{r_{11}}\right]^{1-\alpha[1-\beta_1]} \left[\frac{r_{12}}{r_{12}^a}\right]^{[1-\beta_2][1-\alpha]} \left[\frac{M_{11}^a}{M_1}\right]^{\frac{\alpha}{\sigma-1}} \left[\frac{M_{12}^a}{M_2}\right]^{\frac{1-\alpha}{\sigma-1}}, \\ \frac{U_{12}^a}{U_{12}^f} &= \left[\frac{r_{12}^a}{r_{12}}\right]^{1-[1-\beta_2][1-\alpha]} \left[\frac{r_{11}}{r_{11}^a}\right]^{\alpha[1-\beta_1]} \left[\frac{M_{11}^a}{M_1}\right]^{\frac{\alpha}{\sigma-1}} \left[\frac{M_{12}^a}{M_2}\right]^{\frac{1-\alpha}{\sigma-1}}.\end{aligned}$$

For sufficiently high $\sigma > \overline{\sigma}$, the influence of increase in mass of available varieties will have small effect on ratio of utility levels as $\left[\frac{M_{il}^a}{M_l}\right]^{\frac{\alpha_l}{\sigma-1}}$ would be close to unity. Given, r_{11} increases and r_{12} decreases, we have $\frac{U_{11}^a}{U_{11}^f} < 1$ and $\frac{U_{12}^a}{U_{12}^f} > 1$. ■

Proof of Proposition 6

Proof. Let's introduce notations $A_l = \left[\frac{\phi_{d2l}}{\phi_{x1l}}\left(\phi_{x1l}\right)\left[\frac{f_{xl}}{f_l}\right]^{\frac{1}{\sigma-1}}\tau_l\right]^{\frac{\sigma-1}{\sigma[1-\beta]}}\frac{1}{k_l}$ and $\gamma_{il} = \frac{1}{1+\frac{f_{xl}\Phi(\phi_{xil})}{f_l\Phi(\phi_{dil})}}$.

Similar to free trade case, the relations between the expenditure on labor and sector specific capitals in both countries, the goods market clearing condition (14), the relationship (12) between unit costs across countries in sector 1 and similar relation for sector 2 can be combined to the system of equations (we normalized the w_1 to unity), in which first three equations would be exactly like in the system (29). While last two conditions will modify to

$$\begin{aligned}A_1 r_{11} &= w^{\frac{\beta}{1-\beta}} r_{21} \\ A_2 r_{12} &= w^{\frac{\beta_2}{1-\beta_2}} r_{22}\end{aligned}$$

So that, we have following expression in w:

$$[1-\alpha]\,\frac{1+A_1 w^{-\frac{\beta}{1-\beta}}}{\frac{w}{l}-A_1 w^{-\frac{\beta}{1-\beta}}} = \alpha\frac{1+A_2 w^{-\frac{\beta}{1-\beta}}}{A_2 w^{-\frac{\beta}{1-\beta}}-\frac{w}{l}} \tag{40}$$

Notice that given the symmetry in the endowment structure, we have $w = l = 1$. The inequality similar to the inequality (**35**), for this model will be

$$A_1 w^{-\frac{\beta}{1-\beta}} < 1 < A_2 w^{-\frac{\beta}{1-\beta}}. \tag{41}$$

Also, we have the modified expression for the returns to capital

$$\begin{aligned} r_{11} &= r_1^a \frac{1}{\alpha+[1-\alpha]\frac{1+A_1 w^{-\frac{\beta}{1-\beta}}}{1+A_2 w^{-\frac{\beta}{1-\beta}}}} \\ r_{12} &= r_2^a \frac{1}{[1-\alpha]+\alpha\frac{1+A_2 w^{-\frac{\beta}{1-\beta}}}{1+A_1 w^{-\frac{\beta}{1-\beta}}}} \end{aligned} \tag{42}$$

Let's take complete differential of the left side of equation (**40**). We will get

$$\begin{gathered} \frac{dw}{w}L = \frac{\alpha}{\beta}\left[1+A_2 w^{-\frac{\beta}{1-\beta}}\right]^2 w^{-\frac{\beta}{1-\beta}}\frac{\partial A_1}{\partial\phi_{x11}}d\phi_{x11} \\ +\frac{[1-\alpha]}{\beta}\left[1+A_1 w^{-\frac{\beta}{1-\beta}}\right]^2 w^{-\frac{\beta}{1-\beta}}\frac{\partial A_2}{\partial\phi_{x12}}d\phi_{x12}+\frac{\alpha}{\beta}\left[1+A_2 w^{-\frac{\beta}{1-\beta}}\right]^2 w^{-\frac{\beta}{1-\beta}}\frac{\partial A_1}{\partial\tau_1}d\tau_1 \end{gathered} \tag{43}$$

where L is of following form

$$\begin{gathered} L = \frac{1-\alpha}{\beta}\left[1+[1-\alpha]\frac{1-\beta}{\beta}\right]\frac{\beta}{1-\beta}A_2 w^{-\frac{\beta}{1-\beta}}\left[1+A_1 w^{-\frac{\beta}{1-\beta}}\right]^2+ \\ +[1-\alpha]\frac{\alpha}{\beta}\left[A_1 w^{-\frac{\beta}{1-\beta}}+A_2 w^{-\frac{\beta}{1-\beta}}\right]\left[1+A_1 w^{-\frac{\beta}{1-\beta}}\right]\left[1+A_2 w^{-\frac{\beta}{1-\beta}}\right]+ \\ +\frac{\alpha}{\beta}\left[1+\alpha\frac{1-\beta}{\beta}\right]\frac{\beta}{1-\beta}A_1 w^{-\frac{\beta}{1-\beta}}\left[1+A_2 w^{-\frac{\beta}{1-\beta}}\right]^2 \end{gathered}$$

For every sector we will have expression similar to the expression (**37**). Particularly, we have for sector 1

$$\alpha+[1-\alpha]\frac{1+A_1w^{-\frac{\beta}{1-\beta}}}{1+A_2w^{-\frac{\beta}{1-\beta}}}=\gamma_{11}+[1-\gamma_{21}]\,A_1w^{-\frac{\beta}{1-\beta}} \tag{44}$$

and we have following expression for sector 2

$$[1-\alpha]+\alpha\frac{1+A_2w^{-\frac{\beta}{1-\beta}}}{1+A_1w^{-\frac{\beta}{1-\beta}}}=\gamma_{12}+[1-\gamma_{22}]\,A_2w^{-\frac{\beta}{1-\beta}} \tag{45}$$

We can take the complete differential each of equations (**44**) and (**45**):

$$\begin{gathered}C_1d\phi_{x11}+B_1d\phi_{x12}=\\=\lambda_1\frac{dw}{w}+\left[[1-\alpha]\frac{1}{1+A_2w^{-\frac{\beta}{1-\beta}}}w^{-\frac{\beta}{1-\beta}}\frac{\partial A_1}{\partial\tau_1}-w^{-\frac{\beta}{1-\beta}}\frac{\partial[1-\gamma_{21}]A_1}{\partial\tau_1}\right]d\tau_1\end{gathered} \tag{46}$$

$$\begin{gathered}C_2d\phi_{x11}+A_2d\phi_{x12}=\\=\lambda_2\frac{dw}{w}-\alpha\frac{1+A_2w^{-\frac{\beta}{1-\beta}}}{\left[1+A_1w^{-\frac{\beta}{1-\beta}}\right]^2}w^{-\frac{\beta}{1-\beta}}\frac{\partial A(\phi_{x11})}{\partial\tau_1}d\tau_1\end{gathered} \tag{47}$$

where C_l and B_l are defined as

$$\begin{gathered}C_l=\frac{\partial\gamma_{1l}}{\partial\phi_{x1l}}+\frac{\partial[1-\gamma_{2l}]A_l}{\partial\phi_{x1l}}w^{-\frac{\beta}{1-\beta}}-\alpha_k\frac{\frac{\partial A_l}{\partial\phi_{x1l}}w^{-\frac{\beta}{1-\beta}}}{1+A_kw^{-\frac{\beta}{1-\beta}}},\\B_l=\alpha_k\frac{1+A_lw^{-\frac{\beta}{1-\beta}}}{\left[1+A_kw^{-\frac{\beta}{1-\beta}}\right]^2}\frac{\partial A_k}{\partial\phi_{x1k}}w^{-\frac{\beta}{1-\beta}}.\end{gathered}$$

Also, for λ_i we have following expression:

$$\begin{gathered}\lambda_l=\frac{\beta}{1-\beta}[1-\gamma_{1l}]+\alpha_k\frac{\beta}{1-\beta}\frac{A_lw^{-\frac{\beta}{1-\beta}}-A_kw^{-\frac{\beta}{1-\beta}}}{\left[1+A_kw^{-\frac{\beta}{1-\beta}}\right]^2}A_kw^{-\frac{\beta}{1-\beta}}=\\=\frac{\beta}{1-\beta}[1-\gamma_{2l}]\,A_lw^{-\frac{\beta}{1-\beta}}+\alpha_k\frac{\beta}{1-\beta}\frac{A_kw^{-\frac{\beta}{1-\beta}}-A_lw^{-\frac{\beta}{1-\beta}}}{\left[1+A_kw^{-\frac{\beta}{1-\beta}}\right]^2}\end{gathered}$$

After substituting the expression for dw from condition (**43**) into the expressions and we will get the system of equations in $d\phi_{x11}$ and $d\phi_{x12}$, which can be solved for $d\phi_{x11}$ and $d\phi_{x12}$. For $d\phi_{x11}$ we get

$$d\phi_{x11} = \frac{-G_3 \frac{\partial[1-\gamma_{21}]A_1}{\partial\tau_1} w^{-\frac{\beta}{1-\beta}} + G_2 \frac{\partial A_1}{\partial\tau_1} - G_1 \frac{\partial A_2}{\partial\phi_{x12}} \frac{\partial A_1}{\partial\tau_1}}{G_3 \left[\frac{\partial\gamma_{11}}{\partial\phi_{x11}} + \frac{\partial[1-\gamma_{21}]A_1}{\partial\phi_{x11}} w^{-\frac{\beta}{1-\beta}} \right] - G_2 \frac{\partial A_1}{\partial\phi_{x11}} + G_1 \frac{\partial A_2}{\partial\phi_{x12}} \frac{\partial A_1}{\partial\phi_{x11}}} d\tau_1 \tag{48}$$

where G_1, G_2 and G_3 are positive expressions, given properties 1-5. So, ϕ_{x11} decreases, as τ_1 goes down. Moreover, this result does not depend on sector being abundant with capital. As result, with decrease in variable trade cost in particular sector, the average productivity of firms in this sector increases.

For $d\phi_{x12}$ we have following expression after simplification.

$$\begin{aligned} d\phi_{x12} = \tfrac{1}{\det} \left[\lambda_2 \tfrac{\alpha}{\beta} \tfrac{1}{L} \left[1 + A_2 w^{-\frac{\beta}{1-\beta}} \right]^2 - \alpha \frac{1+A_2 w^{-\frac{\beta}{1-\beta}}}{\left[1+A_1 w^{-\frac{\beta}{1-\beta}} \right]^2} \right] \times \\ \times \left[\frac{\partial\gamma_{11}}{\partial\phi_{x11}} \frac{\partial A_1}{\partial\tau_1} + \left[\frac{\partial[1-\gamma_{21}]}{\partial\phi_{x11}} \frac{\partial A_1}{\partial\tau_1} - \frac{\partial[1-\gamma_{21}]}{\partial\tau_1} \frac{\partial A_1}{\partial\phi_{x11}} \right] A_1 w^{-\frac{\beta}{1-\beta}} \right] w^{-\frac{\beta}{1-\beta}} d\tau_1 \end{aligned} \tag{49}$$

It could be shown that det, the determinant of the system of equations resulted from substituting the expression (**43)** for dw into expressions (**46**) and (**47**), is positive.

Let's show that the expression in the first parenthesis has negative sign. After substituting the expressions for λ_2 and L, following inequality will imply the negative sign of the expression in the first parenthesis

$$
\begin{aligned}
& 1-\gamma_{12} < \\
& < [1-\alpha]\left[1+\tfrac{1-\beta}{\beta}+\alpha\tfrac{1-\beta}{\beta}\tfrac{A_1 w^{-\frac{\beta}{1-\beta}}+A_2 w^{-\frac{\beta}{1-\beta}} A_2 w^{-\frac{\beta}{1-\beta}}}{\left[1+A_1 w^{-\frac{\beta}{1-\beta}}\right] A_2 w^{-\frac{\beta}{1-\beta}}}\right]\times \\
& \times \tfrac{A_2 w^{-\frac{\beta}{1-\beta}}}{1+A_2 w^{-\frac{\beta}{1-\beta}}}+\alpha^2\tfrac{1-\beta}{\beta}\tfrac{A_1 w^{-\frac{\beta}{1-\beta}}\left[1+A_2 w^{-\frac{\beta}{1-\beta}}\right]}{\left[1+A_1 w^{-\frac{\beta}{1-\beta}}\right]^2}+\alpha\tfrac{A_1 w^{-\frac{\beta}{1-\beta}}}{1+A_1 w^{-\frac{\beta}{1-\beta}}}
\end{aligned} \tag{50}
$$

where the right side is bounded from below by $[1-\alpha]\left[1+\frac{1-\beta}{\beta}\right]\frac{A_2 w^{-\frac{\beta}{1-\beta}}}{1+A_2 w^{-\frac{\beta}{1-\beta}}}$. Taking into account inequality (41), we receive

$$
[1-\alpha]\left[1+\frac{1-\beta}{\beta}\right]\frac{1}{2} < [1-\alpha]\left[1+\frac{1-\beta}{\beta}\right]\frac{A_2 w^{-\frac{\beta}{1-\beta}}}{1+A_2 w^{-\frac{\beta}{1-\beta}}}
$$

Given $1-\gamma_{12}(\phi_{x12}) < \frac{1}{2}$, the inequality (50) is satisfied for small enough β. Now, let's show that the expression in second parenthesis of expression (49) is positive.

$$
\begin{aligned}
& \tfrac{\partial\gamma_{11}}{\partial\phi_{x11}}\tfrac{\partial A_1}{\partial\tau_1}+\left[\tfrac{\partial[1-\gamma_{21}]}{\partial\phi_{x11}}\tfrac{\partial A_1}{\partial\tau_1}-\tfrac{\partial[1-\gamma_{21}]}{\partial\tau_1}\tfrac{\partial A_1}{\partial\phi_{x11}}\right]A_1 w^{-\frac{\beta}{1-\beta}} = \\
& = \tfrac{\partial[1-\gamma_{21}]}{\partial\phi_{x11}}\tfrac{\partial A_1}{\partial\tau_1}\left[1-\left[\tfrac{\frac{\partial[1-\gamma_{21}]}{\partial\tau_1}\frac{\partial A_1}{\partial\phi_{x11}}}{\frac{\partial[1-\gamma_{21}]}{\partial\phi_{x11}}\frac{\partial A_1}{\partial\tau_1}}-1\right]A_1 w^{-\frac{\beta}{1-\beta}}\right]
\end{aligned} \tag{51}
$$

Given the cost parameters are the same across sectors, the inequality $k_1 > k_2$ leads to $\phi_{x11} > \phi_{x12}$. So, it could be shown that

$$
\frac{\frac{\partial B_d(\phi_{x11})}{\partial\tau_1}\frac{\partial A(\phi_{x11})}{\partial\phi_{x11}}}{\frac{\partial B_d(\phi_{x11})}{\partial\phi_{x11}}\frac{\partial A(\phi_{x11})}{\partial\tau_1}} < 2
$$

Finally, given $A_1 w^{-\frac{\beta}{1-\beta}} < 1$, we could guarantee that the expression (51) is positive.

We can conclude that for the case, country 1 has more capital in sector 1 than country 2 ($K_{11} > K_{21}$) and the country's endowment of capital in sector 2 is smaller then in country 2 ($K_{12} < K_{22}$), then the decrease in variable trade cost in the sector abundant with capital (sector 1) leads to the increase in other sector (sector 2) exporting productivity cutoff.

Again, the decrease in τ_2 from point of view of country 2 leads to the increase in ϕ_{x21}, as sector 2 is abundant with capital there. According to the system (32), ϕ_{x11} decreases and ϕ_{d11} increases in this case. So, the increase in variable trade cost in sector scarce with capital leads to the increase in other sector exporting productivity cutoff.

As, we can see from equation (43), the decrease in τ_1 has the direct negative effect on w. At the same time, the decrease in ϕ_{x11} caused by the decrease in τ_1 lowers w, while the increase in ϕ_{12} would magnify the direct effect of the decrease in τ_1. Let's demonstrate that the direct effect of the decrease in τ_1 dominates the secondary effect of the decrease in ϕ_{x11}.

Particularly, we should show that the expression in parenthesis

$$\left[\frac{\partial A_1(\phi_{x11})}{\partial \phi_{x11}} \frac{d\phi_{x11}}{d\tau_1} + \frac{\partial A_1(\phi_{x11})}{\partial \tau_1}\right] d\tau_1$$

has positive sign. Using expression (48), for the above expression we get

$$G_3 \frac{\frac{\partial B_x(\phi_{x11})}{\partial \phi_{x11}} \frac{\partial A_1(\phi_{x11})}{\partial \tau_1} + \left[\frac{\partial B_d(\phi_{x11})}{\partial \phi_{x11}} \frac{\partial A_1(\phi_{x11})}{\partial \tau_1} - \frac{\partial A_1(\phi_{x11})}{\partial \phi_{x11}} \frac{\partial B_d(\phi_{x11})}{\partial \tau_1}\right] A_1(\phi_{x11}) w^{-\frac{\beta}{1-\beta}}}{G_3 \left[\frac{\partial B_x(\phi_{x11})}{\partial \phi_{x11}} + \frac{\partial B_d(\phi_{x11}) A_1(\phi_{x11})}{\partial \phi_{x11}} w^{-\frac{\beta_1}{1-\beta_1}}\right] - G_2 \frac{\partial A(\phi_{x11})}{\partial \phi_{x11}} + G_1 \frac{\partial A_2(\phi_{x12})}{\partial \phi_{x12}} \frac{\partial A_1(\phi_{x11})}{\partial \phi_{x11}}} d\tau_1$$

And we have demonstrated that the numerator has positive sign. So, the expression in parenthesis has positive sign.

So, w (equivalently $\frac{w_2}{w_1}$) decrease with the decrease in τ_1. From the point of view of country 2, the decrease in τ_2 leads to the decrease in $\frac{w_1}{w_2}$ or to the increase in w.

With the decrease in τ_1, the right side of equation (45) goes up, as w decreases and ϕ_{x12} increases. The decrease in the left side of this equation leads to the increase in r_{11} and to the decrease in r_{12}, according to the system (36). Similarly, with the decrease in τ_2, the right side of equation (44) go down, as w increases and ϕ_{x12} decreases. The decrease in the left side leads to the increase in r_{11} and to the decrease in r_{12}. So, the effect of the increase in variable trade cost on the rentals on capital does not depend on sector the increase occurred in. $\blacksquare$

REFERENCES

Aw, B., Chung, S. and Roberts, M. (2000) Productivity and turnover in the export market: micro-level evidence from the Republic of Korea and Taiwan (China), *World Bank Economic Review*, **14**, 65-90.

Balassa, B. (1966) Tariff reductions and trade in manufactures among the industrial countries, *American Economic Review*, **56**, 466-473.

Bergstrand, J. (1989) The generalized gravity equation, monopolistic competition, and the factor-proportions theory in international trade, *The Review of Economics and Statistics*, **71**, 143-153

Bernard, A., Eaton, J. and Kortum, S. (2003) Plants and productivity in international trade, *American Economic Review*, **93**, 1268-1290.

Bernard A. and Jensen J. (1999) Exceptional exporter performance: cause, effect, or both?, *Journal of International Economics*, **47**, 1-25.

Bernard, A., Redding, S. and Schott, P. (2007) Comparative advantage and heterogeneous firms, *Review of Economic Studies*, **74**, 31-66.

Bond, E. (1986) Entrepreneurial ability, income distribution, and international trade, *Journal of International Economics,* **20**, 343-356.

Clerides, S., Lach, S., Tybout, J. (1998) Is learning by exporting important? Micro-dynamic evidence from Colombia, Mexico, and Morocco, *Quarterly Journal of Economics,* **113***,* 903-947.

Dixit, A. and Stiglitz, J. (1977) Monopolistic competition and optimum product diversity, *American Economic Review*, **67**, 297-308.

Dornbusch, R., Fischer, S., Samuelson, P. (1980) Heckscher-Ohlin trade theory with a continuum of goods, *Quarterly Journal of Economics*, **95**, 203-224.

Dornbush, R., Fischer, S., Samuelson, P. (1977) Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods, *American Economic Review*, **67**, 823-839.

Eaton, J., Kortum, S. (2002) Technology, geography, and trade, *Econometrica*, **70**, 1741-1779.

Grubel, H. (1967) Intraindustry specialization and the pattern of trade, *Canadian Journal of Economics and Political Science*, **33**, 374-388.

Helpman, E. (1999) The structure of foreign trade, *Journal of Economic Perspectives*, **13**, 121-144.

Helpman, E. and Krugman, P. (1985) Market structure and foreign trade, *Cambbridge: MIT Press.*

Hopenhayn, H. (1992) Entry, exit and firm dynamics in long run equilibrium, *Econometrica*, **60**, 1127-1150.

Hufbauer, C. and Chilas, J. (1974) Specialization by industrial countries: extent and consequences, in Giersch, H. The international division of labor: problems and perspectives.

Jones, R. (1971) A three-factor model in theory, trade and history, In Jagdish Bhagwati et al., eds., Trade, Balance of Payments, and Growth: Papers in International Economics in Honor of Charles P. Kindleberger. Amsterdam: North-Holland.

Krugman, P. (1981) Intraindustry specialization and the gains from trade, *Journal of Political Economy*, **89**, 959-973.

Krugman, P. (1979) Increasing returns, monopolistic competition, and international trade, *Journal of International Economics*, **9**, 469-479.

Mayer, W. (1974) Short and long run equilibria for a small open economy, *Journal of Political Economy*, **82**, 955-967.

Melitz, M. (2003) The impact of trade on intraindustry reallocations and aggregate industry productivity, *Econometrica*, **71**, 1695-1725.

Mussa, M. (1974) Tariffs and the distribution of income: the importance of factor specificity, substitutability, and intensity in the short run and long run, *Journal of Political Economy*, **82**, 1191-1203.

Neary, P. (1978) Short-run capital specificity and the pure theory of international trade, *The Economic Journal*, **88**, 488-510.

Pavcnik, N. (2002) Trade liberalization, exit, and productivity improvements: evidence from Chilean plants, *Review of Economic Studies*, **69**, 245-276.

Romalis, J. (2004) Factor proportions and the structure of commodity trade, *The American Economic Review*, **94**, 67-97.

Stolper, W. and Samuelson, P. (1941) Protection and real wages, *Review of Economic Studies*, **9**, 58-73.